

Exercise 2 – Auction Theory (96573)

1. Formally prove the revelation principle for auctions that have Bayesian-Nash equilibrium.
2. Suppose that there are two players with private values that are independently drawn from the uniform distribution on $[0,1]$.
 - a. Give an exact description of the revenue-maximizing auction for this case.
 - b. Compute the expected revenue of the above auction and of the second-price auction, for this distribution. Which revenue is higher? Explain why this suits the explanations given in class.
3. A **deterministic auction** specifies a winner with probability 1, for every specific realization of v_1, \dots, v_n . A deterministic auction is **monotone** if, when a player wins when bidding v , and she now declares $v' > v$ (and all the others do not change their bids), she continues to win. The **threshold value** of a monotone auction is the infimum over all bids that cause a player to win, given the bids of the other players. Prove: a deterministic auction A is truthful in dominant strategies if and only if (1) A is value-monotone and (2) the winner always pays her threshold value.
4. We have mentioned in class that the virtual-valuation auction maximizes the revenue even if the different players' values are drawn from different distributions. Use Q.3 from above to prove that the virtual-valuation auction is truthful in dominant strategies for this case, and describe the appropriate prices for truthfulness.