Unrepresentative, invalid and misleading: are waiting times for elective admission wrongly calculated?

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Thesis The UK Government Statistical Service reports the percentage of elective 'admissions' that took place in England within 3 months of a patient being added to NHS waiting lists. This percentage is calculated from cross-sectional data using the total number of elective episodes within a specified calendar period as denominator and the number of these enrolled on the waiting list less than 3 months previously as numerator. This approach assumes that NHS waiting lists are closed and stationary populations, and has been widely used by government and non-government researchers in the UK and elsewhere.

Antithesis Little attention has been given to the bias introduced when waiting lists are neither stationary nor closed. This paper identifies four groups of patients which are excluded from the denominator used by the Government Statistical Service and criticises the established method of ignoring left and right censored observations.

Synthesis We describe two alternative formulae that would give the same results as the Government Statistical Service method if waiting lists were closed and stationary, but that also give unbiased results when waiting lists are open and non-stationary. They require a limited amount of additional cross-sectional data to produce upper and lower estimates of the cumulative likelihood of admission among those listed. We recommend the production of unbiased estimates by applying period life-table techniques to a complete and consistent set of 'times since enrolment'.

Keywords waiting lists, cross-sectional studies, stationary population, closed population, selection bias, cohort studies, prospective studies, survival analysis, life tables, state medicine, England.

Introduction
Setting the scene
The UK Government Statistical Service reports the percentage of elective 'admissions' that took place in England within 3 months of a patient being added to the waiting list\(^1\). This percentage is calculated from cross-sectional data using the total number of elective episodes\(^2\) within a specified calendar period as denominator and the number of these enrolled on the waiting list less than 3 months previously as numerator\(^3\). This statistic is used as a measure of the likelihood of elective admission within 3 months of recruitment\(^4\).

The number of elective admissions reflects the likelihood of admission and the numbers 'at risk' of admission within each waiting-time category and calendar period of interest. In other words, the number of elective admissions within the 0–3 month waiting-time category will increase if there is any increase in the likelihood of admission, or in the size of the population exposed to that likelihood. So the size of the numerator accurately reflects conditions within that waiting-time category throughout the period of interest.

But the admissions observed in each waiting-time category are added together to give an indication of the 'extent of exposure' to the 'risk' of elective admission i.e. the data is handled as though it belonged to a cohort followed to extinction, rather than a cross-sectional snapshot. This assumes that the number of patients eligible for elective admission 3–6 months after enrolment is identical to the number surviving admission from the 0–3 month category, although the two groups belong to cohorts of patients which were recruited quite independently. In other words, the existing approach views the waiting list as a closed\(^6\) and stationary\(^7\) population, and only provides an unbiased estimate under these conditions.

Patients, clinicians, managers and politicians all want to know how long people wait for elective admission to hospital. Instead, they are either told about those still waiting\(^8\), or about those already admitted\(^9\). Under no circumstance are they given the whole picture, the likelihood of admission experienced by all those on a waiting list between two calendar dates. The published caveats fail to protect users from equating the likelihoods of admission (within 3, 6, 9, 12 months etc.) among those admitted, with the likelihoods of admission (within 3, 6, 9, 12 months etc.) among those listed. The figures hardly address the question of interest unless this
is the case!

By definition, Hospital episode statistics only collects 'event-based data' and does not capture the waiting times of all those eligible for admission from the waiting list, and this is also true of equivalent approaches elsewhere. But even if we had all the 'times since enrolment' recorded in a treatment registry, we would still have the same problem if the likelihood of elective admission were estimated using only the waiting times of those already admitted. Unless omissions can be viewed as a random sample of the population of all 'times since enrolment', we should expect unrepresentative results. This proposition has received scant attention in the literature.

Clearing the ground
The method currently preferred by the Government Statistical Service reflects firmly-held beliefs about which waiting times count and which do not, i.e. official estimates of the likelihood of admission 'ought' to reflect the experience of those admitted and not the experience of those removed. Exclusion of those removed clearly makes sense when validating waiting lists, we only want to enumerate those who are still eligible for admission at a particular moment in time. But the position is less obvious when we want to measure the 'extent of exposure' which generated elective admissions over a specified calendar period.

Those removed from the waiting list can be divided into two groups by asking whether they were ever really candidates for elective admission. There are those who were never 'at risk' of elective admission and should never have been added to the waiting list. The patient did not want surgery, or the consultant did not agree that an operation was necessary, or had no serious intention of ever calling the patient for surgery. There are also those who were added to the waiting list in good faith, but end up being removed rather than being admitted. The patient's condition may have deteriorated so that the operation is no longer possible, or no longer offers the likelihood of any improvement. They may have died waiting, or have had to have had the operation as an emergency treatment.

The first group of patients is rightly excluded from all the data because they should never have appeared on the waiting list. They should not be enumerated because they were never really eligible for elective admission. They should not contribute to the denominator used to calculate the likelihood of admission because they could not generate admissions. But we would expect the second group of patients to contribute to the data. They were added to the waiting list because they could have been admitted and some may well have been offered a date 'to come in' to hospital. They are only removed from the waiting list because something other than admission intervened (first). These patients should be enumerated until the date they were no longer available. And our assessment of the overall 'extent of exposure' should include that part of their wait which preceded removal from the waiting list. At the time of recruitment to the waiting list it is impossible to distinguish those who will subsequently be admitted, from those who will end up being removed.

Imagine a situation where the first type of patient is never added to the waiting list and where records describing the second type are kept 'up-to-the-minute'. Validation of such waiting lists will not alter the number eligible for admission, despite the fact that some of those enumerated go on to be removed at a later date. This provides an alternative explanation for the disappointing results of such exercises in England. Contrary to expert opinion, these patients should not be deleted from waiting list statistics as though they had never been enrolled.

Aim
This paper describes the limitations of the method currently used to estimate the cumulative likelihood of elective admission within any given time of enrolment on the waiting list. It argues that the existing formula excludes whole categories of patients who might be considered 'at risk' of admission during the period of interest, i.e. it is concerned with the method of calculation, rather than the veracity of the data. Although discussion concentrates on the use made of this approach by the Government Statistical Service in the UK, the method has been widely used by government and non-government researchers in the UK and elsewhere.

Competing events and incomplete observations are omitted
Patients removed from English hospital waiting lists are excluded from subsequent censuses because they are no longer eligible for elective admission. In a similar fashion, those deferred on clinical grounds, or suspended for administrative reasons, are also excluded from census counts for as long as they are not 'at risk' of elective admission. But when we turn our attention to estimating the cumulative likelihood of elective admission within a given time of enrolment, we rely on Hospital episode statistics, which captures data on the understanding that admission must have already taken place. As a result, the waiting times of these three groups of patients are excluded from calculations, as though the individuals had never been added to the waiting list. The experience of those still awaiting elective admission is also discounted, because they cannot appear in Hospital episode statistics during the period of interest.

The matter is further complicated in England because
patients who cancel their admission or fail to arrive as instructed are put to the back of the queue and the date of their enrolment on the list is reset to the date on which they ought to have been admitted. The effect of this is to break the individual's experience into two parts, the first of which ends in self-deferral, or failure to arrive, rather than in admission. Either part may appear in a census, but create particular problems where data-capture depends upon admission to hospital.

- If we measure 'time to admission' from the revised 'date of enrolment', we discount the first part and treat these patients as though they could not have been admitted until after they had been put to the back of the queue. Yet until they self-deferred or failed to attend, these patients were as much 'at risk' of admission as anyone else with the same clinical characteristics and the same length of time on the list. In fact, this was so much the case that they were invited to attend for admission. This approach exaggerates the apparent likelihood of admission within a short time of enrolment.

- An admissions data-set which measures 'time to admission' from the original date of enrolment reports the full length of the patient's experience of the waiting list, but allocates the whole of the second part, and the eventual admission, to the wrong waiting-time categories. This minimises the true likelihood of admission with an official waiting time of less than 3 months.

Estimates of the likelihood of elective admission in England will only reflect day-to-day practice if the two parts are reported separately and if both contribute to the denominator.

Inpatient waiting times are calculated from partial and unrepresentative data; as a result, no one possesses all the relevant facts. Hospital episode statistics omit the exposure to risk contributed by those whose waiting time was incomplete at the close of the period of interest. The only occasion where this will not produce bias occurs when the waiting list can be described as a stationary population. In a stationary waiting list, the number enrolled or reset to zero between 1 July and 30 September 1994 would be the same as the number enrolled or reset to zero over the preceding 92 days, or over the succeeding 92 days (Figure 1). The same equivalence applies to the number 'at risk' of elective admission at the start of every other waiting-time category, i.e. at exactly 3 months, exactly 6 months etc. The number still 'at risk' of admission halfway through each waiting-time category would not change from one census to another. In fact, in a stationary population we would get the same distribution of waiting times, whether we look at groups of patients who were listed together, or groups of patients who were enrolled together. But the hospital waiting list for England would not have attracted so much attention if it were really stationary.

Hospital episode statistics omit the exposure to risk contributed by those whose waiting time ended in some competing event, rather than admission. As a result, the 'times since enrolment' used by the Government Statistical Service are conditional on the fact of admission and over-estimate the likelihood of elective admission experienced by all those on a waiting list between two calendar dates. As there are several ways of curtailing a patient's time on the list, other than elective admission, the hospital waiting list for England is not, in fact, a

![Diagram](image)

**Fig. 1** The English waiting list — the population at risk of being censused on 30 September 1994 and of generating elective admissions in the same waiting-time categories between 1 July and 31 December 1994 inclusive.
closed population.

Event-based data capture waiting times incompletely

A number of these points may be made clearer by use of a lexis diagram. Figure 1 allows us to show an event, such as elective admission, by plotting the date of admission on the horizontal axis and the length of waiting time at admission on the vertical axis. In the same way, a patient's enrolment can be shown by plotting date of enrolment on the horizontal axis. If we join these two points with a diagonal 'life-line', we can read off the length of a patient's experience of the waiting list at any date of interest.

Imagine a patient enrolled on 1 April who failed to attend as instructed 182 days later, on 30 September 1994. The patient was put to the back of the queue on that date and eventually admitted after a further 92 days, on 31 December 1994. This patient would appear in the 0–3 month category of the censuses of 30 June and 30 September, with waiting times of 91 and zero days respectively. The first part of the patient's wait would be reported as ending in a 'failure to attend' during the quarter ending 30 September, while the second part would be described as ending in elective admission during the quarter ending 31 December 1994.

The vertical lines in Figure 1 show censuses conducted just before midnight on 30 June, 30 September and 31 December 1994. The census of the English waiting list counts the number of diagonal 'life-lines' using waiting-time categories which are 3 months wide and which reflect the quarter of enrolment. For example, those recruited to the waiting list in July, August and September 1994 make up the 0–3 month waiting-time category censused at 30 September 1994. The counts at midnight on 30 September 1994 reflect the numbers still 'at risk' of admission halfway through their experience of a waiting-time category.

In Figure 1, each parallelogram contains the group of patients who would be censused in that waiting-time category at midnight, 30 September 1994, if they were still 'at risk' of elective admission. The census counts those who will either survive the waiting-time category, or else be admitted, removed, reset to zero, deferred, or suspended before its close. The census can also be viewed as counting those 'at risk' at the start of a waiting-time category, minus the admissions and competing events that precede the census. An enrolment cohort contributes information on the likelihood of admission from a single waiting-time category during the calendar period of interest.

Hospital episode statistics only collects information on patients who have been admitted and does not capture the waiting times of those removed, or of those who have only been reset to zero, deferred, or suspended. Even if this were otherwise, event-based data capture would still omit incomplete observations. As a result, event-based data capture cannot tell how many people were 'at risk' of admission at the start of a waiting-time category, so the percentage admitted cannot be calculated from Hospital episode statistics alone.

Figure 1 also allows us to distinguish between the waiting times of those listed together and the waiting times of those enrolled together. Patients completing their first 9 months on the waiting list between 1 July 1994 and 31 December 1994 belong to two distinct groups. They are members of the group of patients listed together, or 'at risk' of admission during the calendar period of interest i.e. the 'synthetic' cohort. They are also members of the group of patients enrolled or put to the back of the queue between 1 January and 31 March 1994, i.e. the 'enrolment' cohort. We do not know anything about the initial size of an enrolment cohort, or about the subsequent wait of those newly recruited or put to the back of the queue together, until we reach the calendar period of interest. The members of the enrolment cohort still on the waiting list at 6 months then live through the likelihood of admission experienced by the 6–9 month waiting-time category of the 'synthetic' cohort between 1 July and 31 December 1994. It is clear that the waiting times of patients listed together are sampled from the waiting times of successive cohorts of patients enrolled or put to the back of the queue together.

Non-random exclusions produce unrepresentative results

Bias is introduced wherever random samples are discarded because they give unacceptable results. We might eventually find one which supports our presuppotions but, having used additional criteria to determine which set of results will be reported, we can no longer claim that the sample was selected at random. Nor can we claim to have conducted an independent test of the study hypothesis: the outcome of the trial was a foregone conclusion!

Bias can also be introduced where we discard part of a study population. Imagine we want to verify that a die generates numbers one to six at random. We throw the die and record the number which lands uppermost, repeating the procedure so often that we produce a large and cumbersome set of results. So we discard one-sixth of these to make the data more manageable, but retain all the threes because we are really interested in the probability of throwing a three. As a result, we increase the apparent likelihood of throwing a three from 1/6 to 1/5, even if the die was not loaded.

Discarding data quickly invalidates the results of a
study unless the cases excluded are a random sample of the population recruited. The double-blind randomised controlled trial goes to considerable lengths to avoid the destructive effects of bias. Patients are allocated to treatment and control groups at random, and the study population is preserved from the imposition of additional selection criteria by ‘blinding’. As a result, a patient’s subsequent decision to drop out of the study, or a clinician’s subsequent decision to withdraw a patient, should bear no relation to treatment status in the trial.

We should expect bias wherever patients are excluded from a study at some point after their initial recruitment, unless exclusion occurred at random. The method used by the Government Statistical Service only excludes waiting times that did not end in admission, reducing the size of the denominator without a commensurate reduction in the size of the numerator. Fortunately, this effect is partly offset by inclusion of the left-censored waiting times which ended in admission during the period of interest.

A proportion, an odds, a ratio or the likelihood of admission?

In order to calculate the proportion admitted ($Q_x$) by the end of a waiting-time category ($x, x + n$), we need to know the number ‘at risk’ of admission at the start of the category. This can be estimated as the sum of those ending in admission ($A_x$) or some competing event ($C_x$) plus those surviving the category ($S_x$). ($C_x$ represents all those whose experience of the waiting list ended when they were removed, reset to zero, deferred or suspended from the list during the category.)

$$Q_x = \frac{A_x}{n}$$

If we left $A_x$ out of the denominator in equation (1), we would end up calculating the odds of admission from the category. Although this overestimates the probability of admission, the mistake could be easily corrected using simple algebra. Unfortunately, this is not the case if $C_x$ or $S_x$ are missing; the essential information appears in neither denominator nor numerator and has to be obtained from somewhere else altogether.

The UK Government Statistical Service calculates the proportion of all elective admissions, admitted by the end of a waiting time category, i.e.:

$$Q_x = \frac{A_x}{\sum A_x}$$

$$= \frac{A_x}{\sum A_x + C_x + S_x}$$

This omits competing events ($C_x$) occurring in the category and assumes that the sum of all admissions in ‘subsequent’ waiting-time categories equals the number surviving the category e.g. that $S_0 = A_3 + A_6 + A_9 + A_{12} + A_{15} + A_{18} + A_{21} + A_{24}$. This is untrue. The sum of all ‘subsequent’ admissions observed in the calendar period of interest omits the sum of all ‘subsequent’ competing events. The discrepancy between the denominator used by the UK Government Statistical Service and that proposed in equation (1) may be greater still.

If the size of the waiting list is increasing, the number surviving the category between 1 October and 31 December 1994 will exceed the number surviving the category between 1 July and 30 September 1994 (Figure 2).

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![Figure 2](image-url)
2). Clearly, admissions and competing events are not occurring frequently enough to counter the increase in the numbers enrolled or put to the back of the queue. As a result, the sum of all ‘subsequent’ admissions, plus the sum of all ‘subsequent’ competing events, will underrepresent the number surviving a waiting-time category. The increase in the numbers enrolled or put to the back of the queue will reveal itself by an increase in the numbers ending in admission, or some competing event, in appropriate waiting-time categories after the close of the calendar period of interest.

Unfortunately, little of the information needed by equation (1) is collected by the Government Statistical Service in the UK, although the position can be improved by rearranging its elements to produce equation (3). \( P_x \) represents the number still ‘at risk’ of admission from the category at the time of the census and equals the sum of those surviving the category \( (S_x) \) plus those ending in admission \( (A_x^-) \) or some competing event \( (C_x^-) \) after the census but before the close of the category.

\[
G_x = \frac{n A_x^- + n C_x^-}{n A_x^- + n C_x^- + n P_x}
\]  
(3)

where \( n A_x^- \) and \( n C_x^- \) represent the admissions and competing events that precede the census.

The proportion admitted will only be an accurate estimate of the likelihood of admission if all the admissions from a waiting-time category precede all the competing events \( x \), i.e. if competing events almost fall in the next category. In the absence of any information on the sequence of admissions and competing events, equation (4) estimates the likelihood of admission if all the competing events from a waiting-time category occur so rapidly that they precede all the admissions. In this situation, the ‘competing events’ observed were ‘at risk’ of admission so briefly that they contributed almost nothing to the extent of exposure and can be discounted from this waiting-time category.

\[
G_x = \frac{n A_x^-}{n A_x^- + n C_x^-}
\]  
(4)

The true position lies somewhere between equations (3) and (4).

Conclusions and recommendations
The cumulative likelihood of admission estimated for any given ‘time since enrolment’ depends on how we define the population ‘at risk’ and on how we handle right and left censored waiting times. As a result, published statistics will be biased because they assume that the waiting list is both stationary and closed, and exclude all those not yet, or never to be admitted.

We have no information on the ‘time since enrolment’ of patients removed from the English hospital’s waiting list, reset to zero, deferred, or suspended. Although we measure the exact ‘time since enrolment’ for most elective episodes, we do so using a different definition from the one we apply when enumerating those still awaiting admission. As a result, we cannot carry out a thoroughly satisfactory empirical assessment of the size and direction of bias in published statistics.

We understand that 14% of patients in Australia may expect to be removed from the waiting list for some reason other than admission. If we assume that all these patients belonged to the second of the two groups discussed earlier then clinicians should multiply the published cumulative likelihood for the relevant ‘time since enrolment’ by a factor of 0.86, to estimate the cumulative likelihood which applies to those who are about to join the national waiting list. As there is no reason to believe that the size and direction of bias will be fixed from one waiting list to another, patients, clinicians, managers and politicians should expect existing comparisons of ranked performance to be misleading.

Cumulative likelihoods of elective admission ought to be estimated by applying period life-table techniques to a complete and consistent set of ‘times since enrolment’. This approach could be applied with little further ado to the Swedish National Cataract Register or to the register maintained by the Adult Cardiac Care Network of Ontario. But countries which collect waiting times conditional on the occurrence of an event such as admission (England, Australia and New Zealand) will have to begin by collecting information on the ‘time since enrolment’ of each patient recruited to their waiting list. This information should record the reason why patients were removed from the list and whether the censoring is informative, or non-informative. In the meantime, stable population theory, period life-table techniques and conditional probabilities suggest lines of enquiry that may give some idea of the size of the problem.

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References


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