Service Engineering Spring 2011

Homework 5 - Fluid Models

Note: Some questions are fully solved - these should not be handed in. For other questions, denoted by *, a partial solution is provided - the rest of the solution to these * questions should be submitted. As for the rest of the questions, fully solve them and hand in your solution.

Submit questions: $3, 4, 5, 6, 8, 9^*, 10, 11^*, 12^*, 13$.

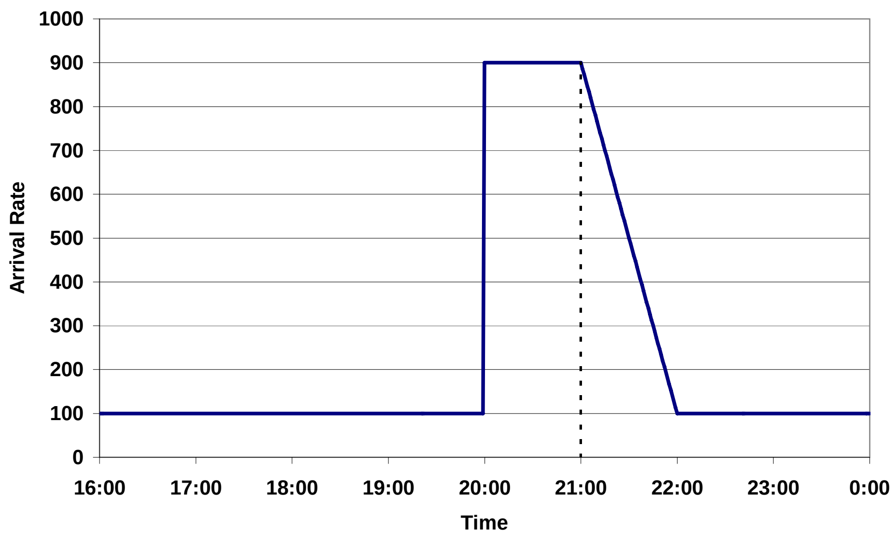
Part 1

Tele-SHOP is a commercial channel dealing with online sales, which is operated by a call-center.

In order to increase profits it was decided to place a short (30 sec) daily advertisement on national TV, at 20:00.

It turns out that the arrivals to the call-center are well approximated by an inhomogeneous Poisson process, with the arrival rate function (customers per hour) given by the figure below:

Figure 1: Arrival rate function



The call center operates from 16:00 till 24:00.

The service duration of each incoming call has an average of 11 minutes. Assume that a working hour of a service representative costs 40 shekel, while a minute of waiting of a customer costs 1.5 shekel.

The cost of running the call center is given by

$$C^{(N)} = \int_0^T \Big[h_t\Big(Q(t) - N(t)\Big)^+ + c_t N(t)\Big]dt,$$

where

- $Q(t)$ - number in system (served + queued) at time t, $\qquad 0 \le t \le T$,
- $N(t)$ - number of servers at time t,
- c_t - staffing cost per unit of time per server[1],
- h_t - waiting cost per unit of time per customer[2].

The system starts empty. In all questions of the homework assume that the **queue must be empty at the end of the day.** Answer the following questions using Excel Solver.

1. **(Solved)** Assume that the number of servers must be fixed during the working day. Find the optimal staffing (i.e. *fixed staffing* that minimizes the costs of running the call center) and calculate the minimal cost.

2. **(Solved)** Solve Question 1. under the assumption that the manager is allowed to change the staffing at the beginning of each hour (*shift staffing*).

3. **(Submit)** From your answer to question 2., it follows that most of the time the staffing level is fixed except for some interval, when it increases. So the manager has decided that he will do the following: in addition to the fixed staffing (call it *basic staffing*), that will work for the whole time interval there will be also an *extra staffing*. The extra staffing will be brought *only* for two hours and will remain *fixed* for these two hours. Help the manager to determine both staffing levels (N_b - basic and N_e - extra) as well as the exact time T_e, when to place the extra staffing.

   Hint: you may find it difficult to apply Excel Solver directly to calculate T_e. Instead of this you may specify the set of possible T_e's (e.g., the beginning of each possible interval) and check them one by one.

From now on assume that upon each service completion, the system gets a reward of r shekel. Assume that waiting customers can abandon the tele-queue and the abandonment rate equals θ (i.e. each waiting customer abandons after waiting an average time of $1/\theta$ **hours**, if not being admitted to service before). In addition, assume that, *instead of waiting costs,* there exists a penalty of s shekel per each abandoning customer. If not stated otherwise, we assume that $s = 11$ shekel and $\theta = 5$.

4. **(Submit)** Write the expression for the total profit of the system.

[1] The units of c_t are monetary units per one time-unit of work of one server (salary).

[2] The units of h_t are monetary units, eg. shekels, per unit of waiting time, i.e. $\frac{\text{shekels}}{\text{waiting customer} \times \text{time unit}}$.

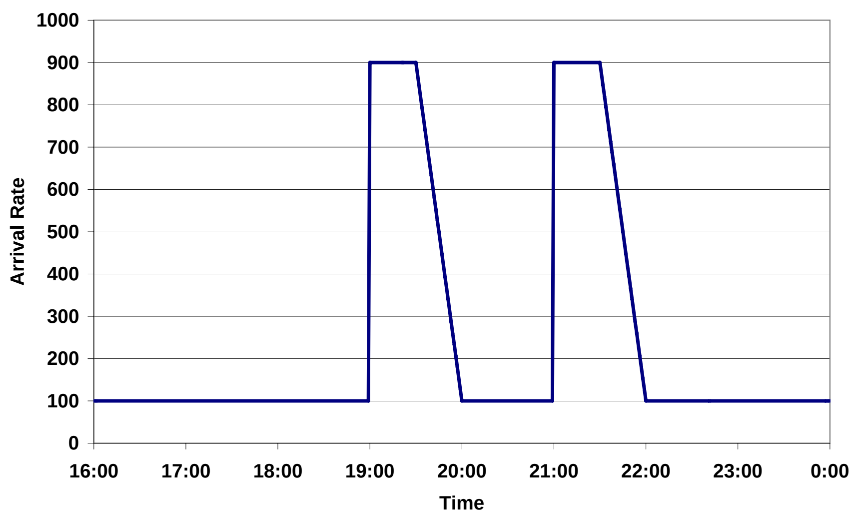
5. **(Submit)** Determine the optimal *fixed* staffing and the maximal profit for $r = 30, 80, 150$. What kind of behavior do you expect from the *fixed* optimal staffing level N^* as r increases to infinity? What is the limit of the optimal staffing level N^* as r increases to infinity?

   Hint: The answer you will get is of the form $\max_{[0,T]} R_t$, where R_t is the solution of a certain differential equation. First state the equation, and then use Excel Solver to find the exact solution. The function R you get here is called the ***offered load*** of the system and it is an analog of $\rho = \lambda/\mu$ for the case when λ is constant in time.

6. **(Submit)** Assume $r = 91$ and $s = 0$ (no penalty for abandoning customers). Compute the optimal *fixed* staffing and the maximal profit. Compare your results with those of Question 5. Explain your findings.

Instead of giving one 30-sec advertisement at 20:00, it was decided to place two 15-sec advertisements at 19:00 and at 21:00. The resulting arrival rate is shown at the figure below. (The arrival rate starts to decrease at 19:30 and 21:30.) Assume $s = 12$ again.

Figure 2: New arrival rate function



7. **(Solved)** For the new arrival rate determine the optimal *fixed* staffing N^* for $r = 80$ and 150. Compare the maximal profits for the two advertising policies. Try to explain your findings.

8. **(Submit)** Assume $r = 80$. Compare the two advertising policies for one-hour shift staffing.

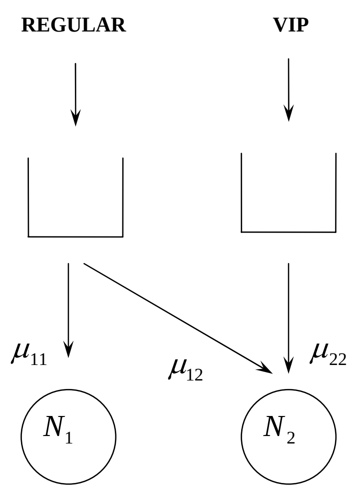
9. **(*)** For the optimal advertising policy and the optimal one-hour shift staffing computed in Question 8., calculate the proportion of customers that abandoned the system without being served.

10. **(Submit)** Assume $r = 25$. Determine the optimal *fixed* staffing and the maximal profit and compute the fraction of abandoning customers. Now add a new performance constraint: the fraction of abandoning customers should not exceed 15%. Find the optimal fixed staffing for the new arrival rate function. How does the new constraint affect the optimal profit?

Part 2. N-model

From now on assume that there are two classes of customers (calls): Regular (class 1) and VIP (class 2), that arrive to the call center. The arrival rates of two classes $\lambda_1(t)$ and $\lambda_2(t)$ are given by Figure 1 (for Regular) and Figure 2 (for VIP). There is no abandonment in the system.
In addition, the call center is virtually divided into two stations 1 and 2 with the corresponding fixed staffing levels N_1 and N_2 (see Figure 3).

Figure 3: The structure of the system.



Servers at station 1 can handle only Regular customers with an average service duration of 20 minutes. Servers at station 2 can handle both classes with an average service duration of 14 minutes for class 1 and 8 minutes for class 2. A working hour of a service representative costs 32 shekel for station 1 and 48 shekel for station 2. A minute of waiting of a customer costs 2 shekel for VIP and 1 shekel for a Regular customer.

Assume for simplicity, that the call center works in the *preemptive-resume regime,* i.e., at every moment a service to a customer can be interrupted (in this case a customer goes back to the queue of its class) and resumed at a later time. Moreover, assume that the VIP customers are *high priority customers*, which means that no Regular customer can be in service at station 2 while VIP customer is waiting.

Define $Q_i(t)$ to be the *total* number of class - i customers in the system, $i = 1, 2$. Let $Q(t) = Q_1(t) + Q_2(t)$. Assume $Q(0) = 0$.

11. **(*)** Assume two operating policies for Regular customers:

Policy 1: If both stations are available to serve Regular customers, they are routed to **Station 1** (both from the queue and from service at Station 2),

Policy 2: If both stations are available to serve Regular customers, they are routed to **Station 2**.

Write the differential equations for $Q_1(t)$ and $Q_2(t)$ for both policies.

12. **(*)** For the two policies, defined in Question 11, determine the optimal fixed staffing for both stations (i.e., two staffing levels N_1^* and N_2^* that minimize the costs of running the call center). Which policy implies lower operating costs?

13. **(Submit)** Consider a different model in which servers at Station 2 can handle only VIP customers, while servers at Station 1 can handle both classes. Assume that servers at Station 1 give priority to VIP customers, and if a VIP customer arrives and there are free servers in both groups, he is will be routed to Station 2. Assume that the service rate of VIP customers is the same in both servers' pools. In addition, as before, we assume for simplicity that the call center works in the *preemptive-resume regime,* plus the regular *work-conserving* assumption. Draw a graph depicting the structure of the system (as done previously in Figure 3). Write the differential equations for $Q_1(t)$ and $Q_2(t)$. Compare the structure defined here with the one defined in Question 11; discuss, when should each structure be used?

General Remarks

1. You may use the Excel file, that was used in the recitations and can be downloaded from the course site, *Recitations* section (Recitation 5). In this file the working day is divided into 480 one-minute intervals and you must use these time intervals.

   Pay Attention !!! All the quantities should be translated to the new time units, for example $\mu = 5$ customers per hour means $1/12$ customers per one-minute interval.

2. The constraint "*queue must be empty at the end of the day*" is true through the whole homework. It turns out that when the above constraint is defined as "*queue **is not positive** at the end of the day*" instead of natural "*queue equals zero at the end of the day*", the Solver works better. Of course, you should input the formulas to the Solver, not the words.

3. You can solve optimization using continuous values of number-of-servers.