

Homework No. 2: Capacity Analysis. Little's Law.

Partial Solution.

Question 4. Since the scientist counts both “going in” and “going out”, the average arrival rate

$$\lambda = \frac{0.061}{2} \text{ insects/hour} = 0.0305 .$$

The average number of insects in the cube $L = 0.0082$. Hence, the average duration of an insect visit

$$W = \frac{L}{\lambda} = 0.269 \text{ hours} = 967.8 \text{ seconds.}$$

Question 5. Arrival rate = total number of customers served. Therefore,

$$\lambda_{90} = 1.12 \cdot \lambda_{89} \quad \text{and} \quad L_{90} = 1.16 \cdot L_{89} .$$

Then

$$\frac{W_{90}}{W_{89}} = \frac{L_{90}}{\lambda_{90}} \cdot \frac{\lambda_{89}}{L_{89}} = \frac{1.16}{1.12} = 103.57\% .$$

So the average duration of a customer visit increased by 3.57% in 1990.

Question 6.

6.1 The average waiting time

$$W = \frac{L}{\lambda} = \frac{L_1 + L_2}{\lambda} = \frac{\lambda_1 W_1 + \lambda_2 W_2}{\lambda} = p_1 W_1 + p_2 W_2$$

is equal to the arithmetic weighted average.

6.2 In this case,

$$\begin{aligned} W &= \frac{L}{\lambda} = \frac{L}{\lambda_1 + \lambda_2} = \frac{L}{L_1/W_1 + L_2/W_2} = \frac{1}{L_1/(L \cdot W_1) + L_2/(L \cdot W_2)} \\ &= \frac{1}{p_1/W_1 + p_2/W_2} \end{aligned}$$

is equal to the harmonic weighted average.

Question 7. There will be employed approximately

$$L = \lambda \cdot W = 100 \cdot 20 = 2000 \text{ accountants.}$$

Assumptions. λ and L will be constant till 2050. The system will reach steady state.

Question 8.

8.1 Estimate the average number of back-logged cases and the average number of cases that the judge resolves per month by

$$\bar{L} = \frac{\sum_{i=1}^{12} L_i}{12} \quad \text{and} \quad \bar{\lambda} = \frac{\sum_{i=1}^{12} \lambda_i}{12},$$

respectively.

Then the average sojourn time of a case in the court per judge is (in months)

$$W = \frac{\bar{L}}{\bar{\lambda}} = \frac{\sum_{i=1}^{12} L_i}{\sum_{i=1}^{12} \lambda_i}.$$

8.2 Let

i = index of month;

j = index of judge.

The arrival rate to judge j is equal to

$$\lambda_j = \frac{1}{12} \lambda_{ij}.$$

The overall arrival rate to the system:

$$\lambda = \sum_j \lambda_j.$$

According to **8.1**, we can estimate the average sojourn time at judge j by

$$W_j = \frac{\sum_{i=1}^{12} L_{ij}}{\sum_{i=1}^{12} \lambda_{ij}}.$$

Now, generalizing **6.1**, we get

$$W = \sum_j \frac{\lambda_j}{\lambda} W_j.$$

Alternative way.

$$W = \frac{\sum_j \sum_{i=1}^{12} L_{ij}/12}{\sum_j \sum_{i=1}^{12} \lambda_{ij}/12} = \frac{\sum_j L_j}{\lambda} = \frac{\sum_j \lambda_j W_j}{\lambda}.$$

Question 10. The ER (emergency room) can be divided into 4 subsystems:

1. Queue for registration.
2. Registration.
3. Queue for doctors.
4. Doctors.

We know that the arrival rate λ to all subsystems is equal to 50 per hour = $5/6$ per minute. Denote by W_i and L_i the waiting time and average number of customers in subsystem i . We know:

$$L_1 = 30; \quad W_2 = 2 \text{ min}; \quad L_3 = 40; \quad W_4 = 5 \cdot 90\% + 30 \cdot 10\% = 7.5 \text{ min}.$$

Now using Little formula for every subsystem we get

$$W_1 = \frac{30}{5/6} = 36 \text{ min}; \quad L_2 = \frac{5}{6} \cdot 2 = \frac{10}{6}; \quad W_3 = \frac{40}{5/6} = 48 \text{ min}; \quad L_4 = \frac{5}{6} \cdot 7.5 = 6.25.$$

If we denote by L and W the average number of customers and the average waiting time in ER respectively, then

$$L = L_1 + L_2 + L_3 + L_4 = 77.9; \quad W = W_1 + W_2 + W_3 + W_4 = 93.5 \text{ min}.$$

Answer. A patient stays 93.5 minutes in the ER, on average. On average, 6.25 patients are being examined by doctors. There are 77.9 patients in the ER, on average.

Remark. The ratio

$$\frac{\text{service time}}{\text{waiting time}} = \frac{2 + 7.5}{36 + 48} = 0.11$$

is very small.