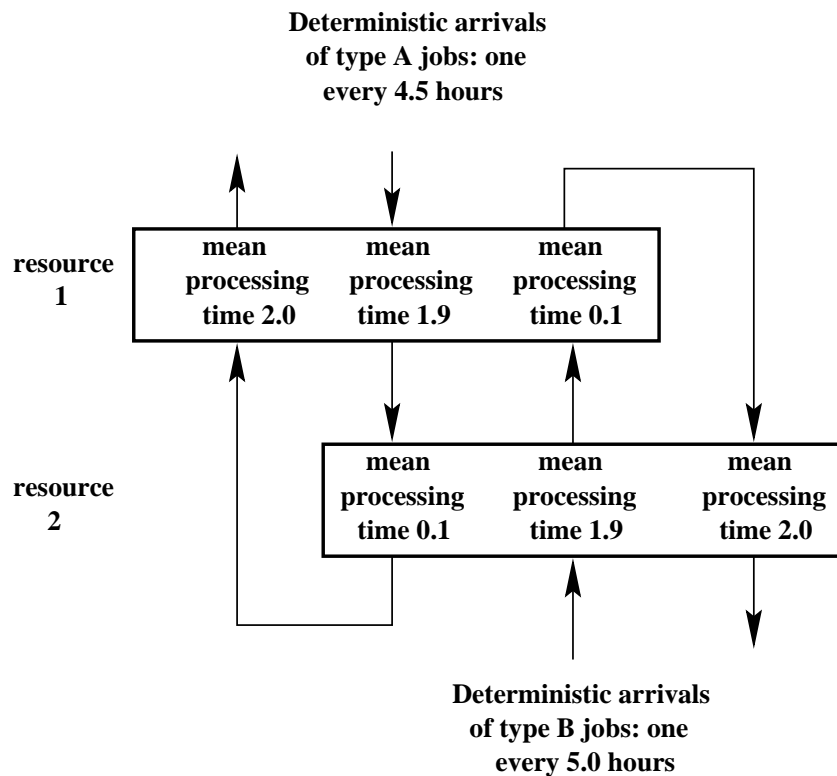


## Homework No. 2: Capacity Analysis. Little’s Law.

Submit questions: 1,3,9,11 and 12.

1. Consider an operation that processes two types of jobs, called type A and type B, and has only two processing resources—you may think of these as operators. The input rates, routing, and average processing times for the two job types are shown in the figure below.

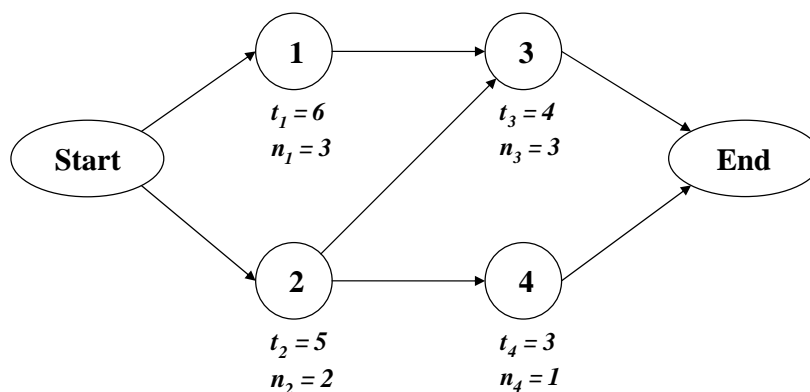


- (a) What is the average utilization of the two processing resources?
- (b) Assume that type A jobs arrived at a rate of one every 3.5 hours. What is the average utilization of resource 1 and resource 2 in this case?

## 2. Tell-On Company

Tell-On company is a supplier of customized telephone switching equipment. The company buys standard electronic modules from outside suppliers, then modifies and assembles them in accordance with customer specifications. The assembly of a Tell-On system may be thought of as a project with four constituent tasks, each performed by a different team of specialists. The precedence constraints among those tasks are expressed by the **precedence** diagram below.

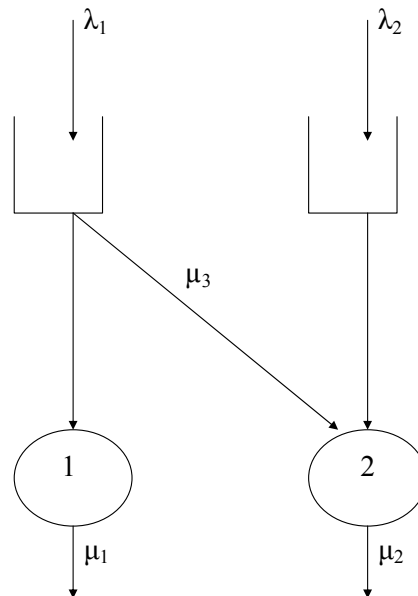
**Note:** For a project to be completed (reach the End state), all of its tasks must be completed. Furthermore, before a task can start, all of its predecessor tasks must be completed. e.g. task 2 must be completed before tasks 3 or 4 may start. (Precedence diagrams are commonly used in PERT/CPM).



- (a) Suppose first that each task  $i$  takes exactly  $t_i$  days (numerical values are shown on the diagram), there is one team devoted to each task, each team begins its task as soon as possible, and there is just one customer order to be dealt with. How long will it take to fill the order? Explain briefly your answer.
- (b) Consider a more interesting dynamic scenario where new customer orders actually arrive according to a Poisson process at an average rate of 0.28 per day. Assume that the system operates in steady-state. Since all tasks must be performed for all projects, the arrival rates of tasks to teams 1-4 are all equal to 0.28 tasks per day. Also, the times required for the tasks involved in completing such orders are independent exponentially distributed random variables, and  $t_i$  is now interpreted as the *expected* time to complete task  $i$ . Finally, there are  $n_i$  different specialist teams dedicated to task  $i$  (these values are shown on the network diagram). Teams execute customer orders on a first-in-first-out basis; each task  $i$  associated with any given order is begun as soon as its predecessor tasks have been completed and there is a qualified specialist team available. The following three questions pertain to this dynamic scenario:
  - (b.1) The average time to complete a customer order (that is, the average response time seen by customers) will be longer than your answer in part (a), perhaps much longer. Explain why. (Do not carry out any calculations; a qualitative answer suffices.)
  - (b.2) What is the utilization for each of the four types of specialist teams? That is, for each type of team, what is the long-run fraction of available team hours that will actually be spent working rather than idle?

- (b.3) Suppose that you could invest in a process improvement that would reduce the average task time for one of the four tasks, leaving everything else unchanged. For which task(s) is such a reduction likely to have the biggest impact on the average response time seen by customers? Why? (Again, provide a qualitative answer.)

### 3. Stability



The figure above describes a model known as the *N-Model*. There are two types of customers, 1 and 2, with average arrival rates 1.135 and 0.38 customers per minute respectively. There are two servers for which the average service rates are: 1 customer per minute for server number one, 1 customer per minute for server number 2, when serving type 2 customers, and 0.5 customers per minute when serving type 1 customers.

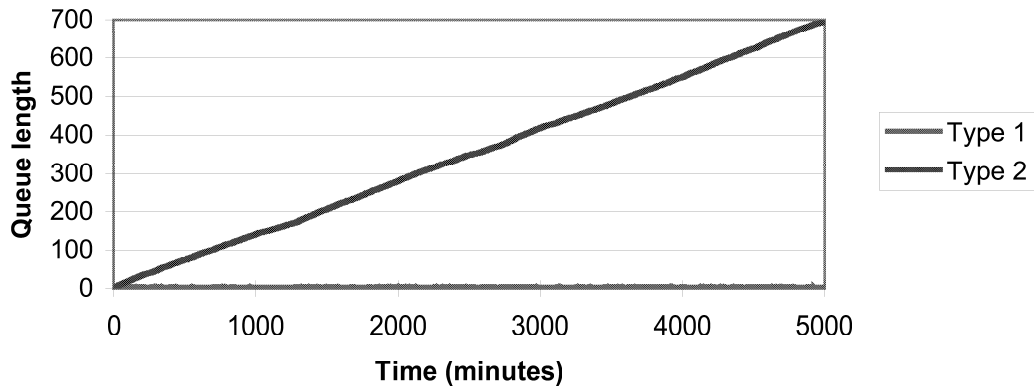
The following scenario could be modeled by the above:

Think of server 2 as a multi-skilled server, capable of serving both types of customers, whereas server 1 specializes in service of type 1 customers, serving them more efficiently than server 2.

Answer the following questions:

- What is the least amount of help that **server 1 has to give to server 2** in order to ensure stability of the system ? i.e. what fraction (%) of the type 1 customers must be served by server number 1 ?
- Suggest a policy that achieves this goal, i.e. a policy that would guarantee that server 1 gives the necessary help to server 2.
- Suppose that type 2 customers are VIP's and type 1 are regulars. What would be a reasonable policy for operating the above system ?

**Remark:** (c) is much harder to answer when customers of type 1 are the VIP's. To see that, consider the following static priority policy: Server 2 serves type 1 customers as long as there are type 1 customers waiting and server 1 is busy. It can be seen in the graph below that with the same system parameters, and under the mentioned reasonable policy, the queue of type 2 would explode:



## Little's Law: $L = \lambda W$

4. A mad scientist has been studying the passage of insects through a certain cubic meter of air in central Minnesota, using automated instruments to continuously monitor insect positions. Her measurements show that, during the calendar year 1990, insects crossed the boundary of the “invisible cube” at an overall rate of 0.061 per hour, **either going in or going out**, and that the average number of insects in the cube was 0.0082. What was the average duration of an insect visit to the cube during 1990?

5. Goldie's Restaurant remains open 24 hours per day, 365 days per year. The total number of customers served in the restaurant during 1990 was 12% greater than the total for 1989. In each year, the number of customers in the restaurant was recorded at a large number of randomly selected times, and the average of those numbers in 1990 was 16% greater than the average in 1989. By how much did the average duration of a customer visit to the restaurant increase or decrease?

6. Consider a service system providing service to two types of customers. Type I customers spend, on average,  $W_1$  units of time in the system and Type II customers spend an average time  $W_2$ . One is interested in calculating the average waiting time  $W$  of the whole population in the following two scenarios:

6.1 We are given (either by experience or through measurements) the quantities  $p_1, p_2$  where  $p_1 = \frac{\lambda_1}{\lambda}$ ,  $p_2 = \frac{\lambda_2}{\lambda}$  are the proportions of customers of the two types within in-flow ( $\lambda = \lambda_1 + \lambda_2$ ,  $\lambda_i =$  in-flow rate of type  $i$ ). Express  $W$  in terms of  $p_i$  and  $W_i$ .

6.2 The quantities  $p_1, p_2$  are given, where  $p_1 = \frac{L_1}{L}$ ,  $p_2 = \frac{L_2}{L}$  are proportions in the system population. ( $L = L_1 + L_2$ ,  $L_i =$  average number in system of type  $i$ .) Express  $W$  in terms of  $p_i$  and  $W_i$ .

7. It is known that 100 candidates on average pass the annual qualification exam for accountants in Israel. An accountant works for 20 years on average (until retirement or professional change). How many accountants will be employed in Israel in 2050? Briefly formulate the assumptions that are used in your solution.

**8.** Assume that  $K$  judges work in Haifa's Labor court of law. The following data was collected for every judge:

**8.1** Number of pending cases that await decision of judge  $j$  ( $1 \leq j \leq K$ ) at the end of the month:

$L_{1,j}, L_{2,j}, L_{3,j}, \dots, L_{12,j}$ , where

$L_{i,j}$  = number of back-logged cases of judge  $j$  at the end of month  $i$ ,  $i = 1, \dots, 12$ .

**8.2** Number of cases that judge  $j$  ( $1 \leq j \leq K$ ) resolved during the months:

$\lambda_{1,j}, \lambda_{2,j}, \lambda_{3,j}, \dots, \lambda_{12,j}$ , where

$\lambda_{i,j}$  = number of cases, resolved by judge  $j$  during month  $i$ ,  $i = 1, \dots, 12$ .

The head-judge would like to estimate the sojourn time of a case in the court, per judge and overall. How can he use the above data without additional measurements? Explain your method and outline restrictions.

**9.** It was mentioned during lecture that many organizations collect only *counting data* ("How many events took place?") without *time data* ("How long were the durations of those events?"). We illustrate this by the article that describes the treatment of cases in the Israel Rabbinate Court. It contains counting data only.

**9.1** Read the article, enclosed on the following page.

**9.2** What is the average sojourn time of a case in the Rabbinate Court? Explain briefly (but rigorously) the assumptions you are using in order to get a reasonable answer.

**9.3** Which additional conclusions concerning performance of the Rabbinate Court can you deduce from the article? For each conclusion, briefly formulate your assumptions.

# חל-אביב מובילה במספר תיקי הגידושין שנפתחו בה בי'95

מתוך 15,874 תיקים שנפתחו בבתי-הדין הרבניים, כשליש  
הוגשו בת"א • העיר מובילה גם במספר הבקשות לשלום-בית

מאת רמי תזות, סגן ידועה אחרונה  
בחל-אביב נפתחו בשנה האחרונה 5,213 תיקי גירושין.  
שי. והיא העיר המובילה בארץ מבחינת מספר תיקי גירושין  
שנפתחו בה בשנת 1995.

בסה"כ נפתחו בשנה האחרונה בבתי הדין הרבניים ברחבי הארץ 15,874 משפטי גירושין. במשך השנה נסגרו 16,577 תיקים (כולל תיקים משנים עברו). הטיפול ב-9,567 משפטי גירושין טרם הסתיים. תל-אביב גם מובילה במספר הבקשות לשלום-בית, 600 מתוך 1,752 שהוגשו בכל רחבי הארץ. השנה האחרונה הביאה עימה בשורה לנשים העגונות. לא פחות מ-108 נשים, שבעליהן נעלמו והותירו אותן עגונות, זכו לגט המיוחל בשנה האחרונה. היחידה המיוחדת לטיפול בנשים עגונות איתרה 29 בלעים באירופה, בארגנטינה ובארה"ב, 10 בעלים בארצות חבר העמים ו-29 בישראל עצמה. לאחר מסעות שכנוע של נציגי הנהלת בתי-הדין הרבניים הסכימו הגברים לתת לנשותיהם גט ולשחרר אותן מעגיונותן רבת השנים. כרגע עדיין עומדים ותלויים 50 תיקים של נשים עגונות. כמו-כן נפתחו בשנה שהסתיימה 1,400 תיקים לגיור. בסך הכל נפתחו בבתי הדין הרבניים בשנה שעברה, 69,743 תיקים, לעומת 71,785 שנסגרו. 16,700 תיקים עדיין פתוחים. את הנתונים מסר אתמול מנהל בתי הדין, הרב אליהו בן דהן. הוא הדגיש, כי בשנת '95 טופלו ונסגרו יותר תיקים מאשר נפתחו, כתוצאה מצעדי התייעלות שנקטים בשנים האחרונות. דהן הזכיר בהקשר זה את מיחשוב בתי הדין, המאפשר מערב מהיר ויעיל אחר התיקים.

Problems 10-12 are adapted from *Managing Business Process Flows*, by R.Anupindi, S.Chopra, S.Deshmukh, J.Van Mieghem, E.Zemel. Prentice Hall, 1999.

**10.** A hospital emergency room (ER) is currently organized so that all patients register through an initial check-in process. At his or her turn, each patient is seen by a doctor and then exits the process, either with a prescription or with admission to the hospital. Currently, 50 people per hour arrive at the ER, 10% of whom are admitted to the hospital. On average, 30 people are waiting to be registered and 40 are registered and waiting to see a doctor. The registration process takes, on average, 2 minutes per patient. Among patients who receive prescriptions, average time spent with a doctor is 5 minutes. Among those admitted to the hospital, average time is 30 minutes. On average, how long does a patient stay in the ER? On average, how many patients are being examined by doctors? On average, how many patients are in the ER?

**11.** A triage system has been proposed for the ER described in Problem 10. As mentioned, 50 patients per hour arrive at the ER. Under the proposed triage plan, patients who are entering will be registered as before. They will then be quickly examined by a nurse practitioner who will classify them as Simple Prescriptions or Potential Admits. While Simple Prescriptions will move on to an area staffed for regular care, Potential Admits will be taken to the emergency area. Planners anticipate that initial examination by a triage nurse will take 3 minutes. They expect that, on average, 20 patients will be waiting to register and 5 will be waiting to be seen by a triage nurse. Recall that registration takes an average of 2 minutes per patient. Planners expect the Simple Prescription area to have, on average, 15 patients waiting to be seen. As before, once a patient's turn comes, each will take 5 minutes of a doctor's time. The hospital anticipates that, on average, the emergency area will have only 1 patient waiting to be seen. As before, once that patient's turn comes, he or she will take 30 minutes of a doctor's time. Assume that, as before, 90% of all patients are Simple Prescriptions. Assume, too, that triage nurses are 100% accurate in their classifications. Under the proposed plan, how long, on average, will a patient stay in the ER? On average, how long will a Potential Admit stay in the ER? On average, how many patients will be in the ER? What is the least number of nurses needed to accommodate the load?

**12.** Refer again to Problems 10 and 11. Once the triage system is put in place, it performs quite close to expectations. All data conform to planners' expectations except for one set—the classifications made by the nurse practitioner. Assume that the triage nurse has been sending 91% of all patients to the Simple Prescription area when in fact only 90% should have been so classified. The remaining 1% are discovered when transferred to the emergency area by a doctor. As a result, 19 patients, on average, are waiting in the Simple Prescription area (versus 15 patients in Problem 11). Assume that wrongly classified patients spend, on average, 10 minutes at their first doctor (Simple Prescriptions area) and 30 minutes at their second doctor in the emergency area. In Addition, notice that the wrongly classified patients visit two queues at the last stage. Assume all other information from Problem 11 is valid. On average, how long does a patient stay in the ER? On average, how long does a Potential Admit stay in the ER? On average, how many patients are in the ER?