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Provision of a public good with bounded cost

Nabil I. Al-Najjar^{a,*}, Rann Smorodinsky^b

^a*Department of Managerial Economics and Decision Sciences, Kellogg School of Management, Northwestern University, Evanston, IL 60208, USA*

^b*Davidson Faculty of Industrial Engineering and Management, Technion, Haifa 32000, Israel*

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Abstract

We provide a sufficient condition for the expected aggregate contribution to a public good to be bounded, independently of the size of the population. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

One of the earliest and most basic insights of the public goods literature is the effect of population size on the severity of the free-rider problem. Rob (1989) and Mailath and Postlewaite (1990) formalized this intuition using models with private information and increasing number of agents. More specifically, consider a public good economy \mathcal{E}_N with N individuals, each having private information about his valuation for a public project. The project costs $C_N \geq \beta N$, where $\beta > 0$ is a lower bound on the per capita cost of the project. Mailath and Postlewaite (1990) showed that if there is uncertainty about individuals' valuations, the probability of provision of the public good goes to zero, uniformly over all incentive compatible, individually rational, balanced mechanisms.

Such results may be interpreted as saying that the maximum expected *per capita* contribution voluntarily collected from a population decreases to zero as population size grows. Thus, although the *aggregate* contribution may be unbounded in N , its rate of increase eventually falls short of covering the linearly increasing cost of the project.

In practice, many public projects have costs that are either fixed, or grow very slowly in N . One

*Corresponding author. Tel.: +1-847-491-5426; fax: +1-847-467-1220.

E-mail address: al-najjar@nwu.edu (N.I. Al-Najjar)

class of examples includes investments in information- or knowledge-based projects (basic technological research, medical research and so on) whose costs are fixed independently of the size of the population benefiting from these projects. In this important class of public good problems, existing asymptotic impossibility results are, in principle, consistent with solutions based on voluntary contribution schemes.

This paper is an attempt to address the question of what makes it possible to extract an unbounded aggregate contribution when — as we know from earlier literature — the per capita contribution goes to zero. Unbounded aggregate contributions can, in principle, result from either: (1) many individuals making vanishingly small contributions as N goes to infinity; or (2) individuals in a subset K_N each makes a large contribution, with K_N unbounded, but $K_N/N \rightarrow 0$.

We show that the latter is not possible: for any threshold $\alpha > 0$, the number of individuals whose expected contribution exceeds α is bounded by a number K_α that is independent of N and the mechanism being played (provided it satisfies IR and IC). The only way for a mechanism to generate an unbounded aggregate contribution is to collect vanishingly small contributions from a vast number of individuals. From this it follows that, within the class of mechanisms requiring a minimum expected contribution (due to administrative collection costs, say), expected aggregate contribution is bounded independently of the size of the community and the particular mechanism used.

2. The model

We consider a class of *public good economies*. Each economy \mathcal{E}_N is parametrized by the number of individuals N and satisfies the following assumptions. Individual n 's valuation, or type, is a random variable t_n taking values in a finite set $T_n \subset [0, t^+]$, which contains 0 for all n . Let $T = \prod_n T_n$ denote the set of type profiles and P the joint distribution of types. We assume that valuations are independent and that there is $\epsilon > 0$ such that $P(t_n) \geq \epsilon$ uniformly across n , t_n , and \mathcal{E}_N .

Contributions are collected via mechanisms that determine the probability of provision and contributions as function of agents' reported types. More specifically, there are two possible collective outcomes corresponding to whether the project is undertaken or not. A mechanism is a pair (δ, c) , where $\delta: T \rightarrow [0, 1]$ is the probability of provision as a function of the reported type profiles, and c is a vector of contributions (c_1, \dots, c_N) , where $c_n: T \rightarrow R$ denotes the amount contributed by individual n . Using the revelation principle, we restrict attention to direct revelation mechanisms in which each agent truthfully reports his type. Individual n 's payoff under (δ, c) in this case is:¹

$$u_n = t_n \delta - c_n.$$

We require (δ, c) to be interim *individually rational*:

$$E(u_n | t_n) \geq 0 \quad \text{for all } n \text{ and } t_n, \tag{IR}$$

and *incentive compatible*: for every n , t_n and $t'_n \in T_n$

¹This formulation covers mechanisms which allow partial or total reimbursement of contributions if the project is not built. For example, the case of full reimbursement can be expressed by requiring $c_n = 0$ whenever $\delta = 0$.

$$t_n E(\delta|t_n) - E(c_n|t_n) \geq t_n E(\delta|t'_n) - E(c_n|t'_n) \tag{IC}$$

Later we introduce a *budget balance* constraint. For the moment, however, our concern is the (expected) aggregate contribution, $E \sum_n c_n(t)$, generated by a mechanism.

3. Main result

The key assumption we add to the standard requirements of IR and IC is that the mechanism (δ, c) satisfies *No Small Contributors* condition, relative to a parameter $c^- > 0$: for every n

$$E(c_n) < c^- \Rightarrow c_n(\mathbf{t}) = 0 \quad \text{for all } \mathbf{t}. \tag{NSC}$$

This says that if the expected amount the mechanism collects from individual n of type t_n falls below the threshold c^- , then the mechanism completely ignores this individual — it collects nothing from him.

This assumption, while restrictive, is relevant in many public good environments. One interpretation is that the mechanism prepares a ‘list of active contributors’ and that contributions are collected only from those on that list. The decision to include or exclude an individual is based on ex ante available information — in this case, the joint distribution P and the mechanism — but not on information known only ex post after the entire type profile is revealed. Suppose there is a fixed cost (e.g., of administrative nature) for including an individual in the list. Assumption NSC then says that the mechanism will not include an individual if his expected contribution falls below a threshold c^- .

The next result shows that NSC makes it impossible to extract an unbounded aggregate contribution:

Proposition 1. *For every $\epsilon > 0$, there is a constant $C > 0$, independent of N , such that in any public good economy \mathcal{E}_N ,*

$$\sup_{(\delta, c)} \sum E c_n \leq C$$

where the sup is taken over all mechanisms satisfying IR, IC, and NSC.

The formal proof builds on our earlier work characterizing the number of pivotal players in a game. Fix a public good economy \mathcal{E}_N , and for any function $f: T \rightarrow [0, 1]$, define

$$V_n(f) = \max_{t \in T_n} E(f|t) - \min_{t' \in T_n} E(f|t').$$

This represents the maximum effect individual n can have on the expectation of f . For $\alpha > 0$, call individual n α -pivotal relative to f if $V_n(f) \geq \alpha$.

The following adapts Theorem 1 in Al-Najjar and Smorodinsky (1996) to our context, providing the main technical device we use in the proof of Proposition 1.

Proposition 2. *Fix $\epsilon, \alpha > 0$. Then there is K^*_α , depending only on ϵ and α , such that the number of α -pivotal individuals does not exceed K^*_α uniformly over N, \mathcal{E}_N and f .*

Proof of Proposition 1. Consider an arbitrary mechanism (δ, c) satisfying the conditions of Proposition 1. The incentive compatibility constraint for a type t_n misrepresenting his type to be $t_n = 0$ is

$$t_n E(\delta|t_n) - E(c_n|t_n) \geq t_n E(\delta|t_n = 0) - E(c_n|t_n = 0).$$

Using IR for type $t_n = 0$, we have that $E(c_n|t_n = 0) \leq 0$. Rearranging terms, we have

$$E(c_n|t_n) \leq t_n [E(\delta|t_n) - E(\delta|t_n = 0)] \leq t_n V_n(\delta).$$

That is, agent n 's expected contribution is bounded by the influence of his report on the probability of provision. Note that $V_n(\delta)$ is not a random variable.

Taking expectations over t_n , we have

$$E(c_n) \leq V_n(\delta) E t_n.$$

Fix

$$\alpha = \frac{c^-}{\max_n E t_n}.$$

By Proposition 2, there are at most K^*_α individuals \mathcal{E}_N for whom $V_n(\delta) \geq \alpha$. The maximum expected contribution of an α -pivotal individual N is $E t_n$, so the total contribution of all such individuals cannot exceed $[\max_n E t_n] K^*_\alpha$. On the other hand, the remaining non-pivotal agents have expected contribution not exceeding c^- , so by condition NSC they contribute zero at all type profiles. Thus, the expected aggregate contribution cannot exceed $[\max_n E t_n] K^*_\alpha < t^+ K^*_\alpha$. ■

4. Example and discussion

1. *Failure of provision:* So far we have focused on the maximum expected aggregate contribution to a public project. Proposition 2 has an obvious implication for the probability of provision of a project of fixed cost C when a budget balance condition is imposed:

$$CE\delta \leq E \sum_n c_n(t). \tag{BB}$$

This is a weak form of budget balancing, requiring only the (ex ante) balance between expected expenditures and expected contributions.² A direct consequence of the proof of Proposition 2 is:

Proposition 3. For any population size N and public good economy \mathcal{E}_N , if (δ, c) satisfies IR, IC, BB, and NSC, then

$$E\delta \leq \frac{[\max_n E t_n] K^*_\alpha}{C} \tag{a}$$

²This may be justified by imagining that the community has access to a risk neutral credit market (see Mailath and Postlewaite (1990) for discussion). Note that the result holds a fortiori if stricter budget balance conditions are imposed.

For every $\epsilon > 0$, there is \bar{C} such that $C > \bar{C}$ imply that $E\delta < \epsilon$. (b)

In particular, $E\delta \rightarrow 0$ as $C \rightarrow \infty$, uniformly in population size.

2. *Numerical example:* the analysis in Al-Najjar and Smorodinsky (1996) provides an asymptotic estimate $K^*_\alpha \simeq 1/\epsilon\pi\alpha^2$. This makes it possible to use Proposition 2 to compute numerical bounds on aggregate contribution. For instance, suppose that $\epsilon = 0.10$, $c^- = 1$, and $\max_n Et_n = 10$. Then $\alpha = 1/10 = 0.1$, so $K^*_\alpha \simeq 1/(0.1) \cdot (3.14) \cdot (0.1^2) \leq 320$.

Proposition 2 then implies that expected aggregate contribution never exceeds \$3200. From Proposition 3, the probability of building a project costing \$32 000, say, never exceeds 1% regardless of population size. This is so, even though, in a large population, there is probability almost 1 that aggregate valuations exceed the cost of the project.

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