Partial Transmit Sequences for Peak-to-Average Power Ratio Reduction of OFDM Signals With the Cross-Entropy Method

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***Abstract*—This letter considers the use of the partial transmit sequence (PTS) technique to reduce the peak-to-average power ratio (PAPR) of an orthogonal frequency division multiplexing (OFDM) signal. The conventional PTS technique can provide good PAPR reduction performance for OFDM signals; however, it requires an exhaustive search over all combinations of allowed phase factors, resulting in high complexity. In order to reduce the complexity while still improving the PAPR statistics of an OFDM signal, a new method using the Cross-Entropy (CE) method is proposed to reduce both the PAPR and the computational load. In the proposed CE method, we first define a score or fitness function based on the corresponding PAPR reduction performance. The score function is then translated into a stochastic approximation problem which can be solved effectively. The simulation results show that the performance of the proposed CE method provides almost the same PAPR reduction as that of the conventional exhaustive search algorithm while maintaining low complexity.**

***Index Terms*—Cross-entropy method, orthogonal frequency division multiplexing, partial transmit sequence, peak-to-average power ratio.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive technique for achieving high-bit-rate wireless data communication [1]. It has been applied extensively to digital transmission, such as in wireless local area networks and digital video/audio broadcasting systems. Moreover, it has been regarded as a promising transmission technique for fourth-generation wireless mobile communications. However, due to its multicarrier nature, the major drawback of the OFDM system is the high peak-to-average power ratio (PAPR), which may cause high out-of-band radiation when the OFDM signal is passed through a radio frequency power amplifier. Consequently, the high PAPR is one of the most important implementation challenges that face designers of OFDM.

Various methods for PAPR reduction have been proposed in the literature [2]–[6] to avoid the occurrence of high PAPR of OFDM signals. Among these methods, the partial transmit sequence (PTS) technique [6] is the most attractive scheme because of good PAPR reduction performance and no restrictions to the number of the subcarriers. In the PTS scheme, the input data is divided into smaller disjoint subblocks. Each subblock is multiplied by rotating phase factors. The subblocks are then added to form the OFDM symbol for transmission. The objective of the PTS scheme is to design an optimal phase factor for a subblock set that minimizes the PAPR.

The PTS technique significantly reduces the PAPR, but unfortunately, finding the optimal phase factors is a highly complex problem. In order to reduce the search complexity, the selection of the phase factors is limited to a set of finite number of elements. The exhaustive search algorithm (ESA) is then employed to find the best phase factor. However, the ESA requires an exhaustive search over all combinations of the allowed phase factors and has exponential search complexity with the number of subblocks.

To reduce the computational complexity, some simplified search techniques have recently been proposed [7]–[9], such as the iterative flipping algorithm (IFA) [9]. Although the IFA significantly reduces the search complexity, there is some gap between its PAPR reduction performance and that of the ESA. In general, for all these search methods, either the PAPR reduction is suboptimal or the complexity is still high. In this paper, we consider using the PTS technique to reduce the PAPR of the OFDM signals and propose a novel implementation of the PTS technique to reduce the PAPR based on the Cross-Entropy (CE) method [10]. In the proposed CE method, we first define the amount of the PAPR as the score function. The score function is then translated into a stochastic approximation problem which can be solved effectively. The simulation results show that the performance of the proposed CE method provides almost the same PAPR reduction as that of the ESA while maintaining low complexity.

II. OFDM System Model and Problem Definition

A. OFDM System Model and PAPR Definition

In an OFDM system with N subcarriers, the discrete-time transmitted OFDM signal is given by

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nk/PN}, \quad k = 1, 2, \ldots, PN-1 \quad (1)$$

where $j = \sqrt{-1}$, X_n, $n = 0, 1, \ldots, N-1$, are input symbols modulated by PSK or QAM, and P is an integer

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that larger or equal 1 called over-sampling factor. When $P = 1$, the samples are achieved by use of the Nyquist rate sampling. We shall write the input data block as a vector, $\mathbf{X} = [X_0 \quad X_1 \quad \cdots \quad X_{N-1}]^T$. The PAPR of the transmitted signal in (1), defined as the ratio of the maximum to the average power, can be expressed by

$$\mathrm{PAPR} = 10\log_{10}\frac{\max|x_k|^2}{E\left[|x_k|^2\right]}\ (\mathrm{dB}) \tag{2}$$

where $E[\cdot]$ denotes the expected value operation.

B. OFDM System With PTS to Reduce the PAPR

As shown in Fig. 1, in a typical OFDM system with a PTS scheme to reduce PAPR, the input data $\mathbf{X}$ is partitioned into smaller M disjoint subblocks, which are represented by the vector $\mathbf{X}_m$, where $m = 1, 2, \ldots, M$, such that

$$\mathbf{X} = \sum_{m=1}^{M} \mathbf{X}_m. \tag{3}$$

Here, it is assumed that each subblock consists of a set of subcarriers of equal size. Next, the partitioned subblocks are converted from the frequency domain to the time domain using the N-point inverse fast Fourier transform (IFFT). Since the IFFT is a linear transformation, the representation of the block in time domain is given by

$$\mathbf{x} = IFFT\left\{\sum_{m=1}^{M}\mathbf{X}_m\right\} = \sum_{m=1}^{M} IFFT\{\mathbf{X}_m\} \triangleq \sum_{m=1}^{M}\mathbf{x}_m. \tag{4}$$

Then, the time domain sequences are combined with complex phase factors $\mathbf{b} = [b_1 \quad b_2 \quad \cdots \quad b_M]^T$ to minimize the PAPR. That is, the PAPR is reduced by the weighted combination of M subblocks. The resulting time domain signal after combination is given by

$$\mathbf{x}'(\mathbf{b}) = \sum_{m=1}^{M} b_m \cdot \mathbf{x}_m. \tag{5}$$

The allowable phase factors are

$$b_m = e^{j\phi_m}, \tag{6}$$

where ϕ_m can be chosen freely within $[0, 2\pi)$. For convenience, (5) can be expressed as

$$\mathbf{x}'(\Phi) = \sum_{m=1}^{M} e^{j\phi_m}\cdot\mathbf{x}_m, \tag{7}$$

where $\Phi = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_M]^T$. Hence, the objective of the PTS scheme is to design an optimal phase factor for the subblock set that minimizes the PAPR. Thus, the minimum PAPR with the PTS technique is related to the problem

$$\begin{aligned}&\text{minimize} \quad \max|\mathbf{x}'(\Phi)|\\ &\text{subjectto} \quad 0 \le \phi_m < 2\pi, \quad m = 1, 2, \ldots, M.\end{aligned} \tag{8}$$

It is obvious that finding a best phase factor set is a complex and difficult problem; therefore, in the next section, we propose a novel implementation of the PTS scheme based on the CE method.

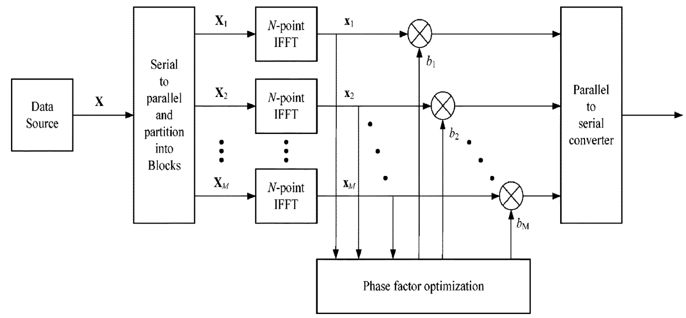

Fig. 1. Block diagram of the partial transmit sequence approach.

III. The Cross-Entropy Method and its Application to PTS to Reduce PAPR

A. The CE Method

The CE method was first proposed by Rubinstein [10] to solve rare event estimation problems and was soon successfully applied to solve both combinatorial and continuous optimization problems. The CE method is a general algorithm that is used to solve global optimization tasks of the form

$$\arg\max_{\boldsymbol{\theta}\in\Theta} S(\boldsymbol{\theta}). \tag{9}$$

That is, we wish to maximize the score function $S(\boldsymbol{\theta})$ over all $\boldsymbol{\theta}$ in set Θ. Instead of maintaining a simple solution candidate $\boldsymbol{\theta}_l$ in each time step for the conventional optimization algorithms, the main idea of the CE method is to maintain a distribution of possible solutions and adaptively update this distribution according to the Kullback–Leibler distance, i.e., cross entropy, between the associated density and the optimal importance sampling density. By doing so, one constructs a random sequence of solutions which converges (probabilistically) to the optimal or, at the least, to a reasonable, solution. In short, the CE method involves the following two iterative phases: 1) generating random samples in Θ according to a specified sampling distribution generated from the previous iteration and 2) updating the parameters on the basis of the best scoring samples in order to produce better scoring samples in the next iteration. For a concrete understanding of the CE method, the reader is referred to [10].

B. The CE Algorithm Based PTS Technique

In the PTS approach, the objective is to find the phase factors with the aim of minimizing the PAPR. However, in order to employ the CE method to find the phase factors that minimize the PAPR in the PTS technique, we have to define the score function for the proposed CE algorithm. The score function, defined as the amount of PAPR reduction, can be expressed as

$$L(\mathbf{x}'(\Phi)) = \frac{1}{10\log_{10}\frac{\max|\mathbf{x}'(\Phi)|^2}{E\left[|\mathbf{x}'(\Phi)|^2\right]}}. \tag{10}$$

That is, we define the inverse of the PAPR as the evaluation function such that its value increases as the PAPR decreases. Hence, in the proposed CE algorithm-based PTS approach, we are interested in maximizing the score function, expressed in (10), over the set $[0, 2\pi)$ for all potential Φ such that

$$\arg \max_{\Phi \in [0,2\pi)} L\left(\mathbf{x}'\left(\Phi\right)\right). \tag{11}$$

The CE method is an adaptive importance sampling method that transforms the deterministic optimization problem (11) into a family of stochastic sampling problems. Hence, the first step in using the CE method is to randomize our original deterministic problem (11) by including a set of sampling distribution over deterministic Φ. In this paper, we take the sampling distribution to be a Gaussian distribution $N\left(\alpha, \beta\right)$, where α and β are the mean and variance, respectively. It is important to emphasize that the sampling distribution can be quite arbitrary and does not need to be related to the function that is being optimized. The reason we adopt the Gaussian distribution is that it gives formulas that are easy to update

At each iteration l of the algorithm, a collection of K random samples $\left\{\left\{\phi_m^{(l)}\left(k\right)\right\}_{m=1}^{M}\right\}_{k=1}^{K}$ is obtained from a Gaussian distribution, i.e., $\phi_m^{(l)}\left(k\right) \sim N\left(\alpha_m^{(l)}, \beta_m^{(l)}\right)$, where $\phi_m^{(l)}\left(k\right)$ denotes the m^{th} element of the sample k at iteration l, and $\alpha_m^{(l)}$ and $\beta_m^{(l)}$ denote the mean and the variance of the m^{th} element at iteration l, respectively. Next, with the K samples, we can derive a set of performance values $\left\{L\left(\Phi^{(l)}\left(k\right)\right)\right\}_{k=1}^{K}$ using the score function $L\left(\Phi\right)$ expressed in (10). After sorting the performance values from the smallest to the largest, $L_{(1)} \leq \cdots \leq L_{(K)}$, we select the best few of the performance values that exceed a certain level to update new values $\alpha_m^{(l)}$ and $\beta_m^{(l)}$. That is, $\alpha_m^{(l)}$ and $\beta_m^{(l)}$ are updated via the sample mean and sample standard deviation of a fixed number of the best performance samples, the so-called elite samples $K^{\text{elite}} = \lceil \rho K \rceil$, where ρ denotes the fraction of the best samples and $\lceil \rho K \rceil$ is the integer part of ρK. Let χ be the indices of the K^{elite} best performance samples. Then $\alpha_m^{(l)}$ and $\beta_m^{(l)}$ can be updated via

$$\tilde{\alpha}_m^{(l)} = \sum_{k \in \chi} \frac{\phi_m^{(l)}\left(k\right)}{K^{\text{elite}}} \tag{12}$$

and

$$\tilde{\beta}_m^{(l)} = \sum_{k \in \chi} \frac{\left(\phi_m^{(l)}\left(k\right) - \alpha_m^{(l)}\right)^2}{K^{\text{elite}}} \tag{13}$$

respectively. That is, we update $\alpha_m^{(l)}$ and $\beta_m^{(l)}$ as the mean and variance of the elite samples. In addition, instead of using $\tilde{\alpha}_m^{(l)}$ and $\tilde{\beta}_m^{(l)}$ as the updated parameters, it is beneficial to add a smoothing procedure for each iteration l as

$$\alpha_m^{(l)} = \gamma \tilde{\alpha}_m^{(l)} + \left(1 - \gamma\right) \alpha_m^{(l-1)} \tag{14}$$

and

$$\beta_m^{(l)} = \gamma \tilde{\beta}_m^{(l)} + \left(1 - \gamma\right) \beta_m^{(l-1)} \tag{15}$$

where γ is the smoothing parameter, with $0 < \gamma \leq 1$. It is obvious that for $\gamma = 1$, we have the original updating rule. The main algorithm is summarized as follows:

Algorithm: Employ the CE algorithm to search the optimal phase factor for the PTS technique to reduce the PAPR

1) Initialize $\alpha_m^{(0)}$ and $\beta_m^{(0)}$. *Set* $l = 1$.
2) Use the density $N\left(\alpha_m^{(l-1)}, \beta_m^{(l-1)}\right)$ to generate a random sample $\left\{\left\{\phi_m^{(l)}\left(k\right)\right\}_{m=1}^{M}\right\}_{k=1}^{K}$.
3) Calculate the score function according to (10) to get a set of performance values $\left\{L\left(\Phi^{(l)}\left(k\right)\right)\right\}_{k=1}^{K}$.
4) Order the performance values from the smallest to the largest, and select the best K^{elite} elite performance values according to the predetermined quantile parameter ρ.
5) Calculate the sample mean and sample variance of the elite samples according to (12) and (13), respectively.
6) Update the mean and variance of the elite samples in a smooth way, as in (14) and (15), respectively.
7) Repeat step 2 to step 6 for $l = l + 1$ until the stop criterion is met.

Next, if the selections of the allowed phase factors are limited to a set of finite number of elements, we propose a modified CE algorithm (MCE). The MCE algorithm is similar to the original CE (OCE) algorithm described above except that we insert an additional step between the step 2 and the step 3 of the OCE algorithm. The function of the additional step is to map each random sample $\phi_m^{(l)}\left(k\right)$ into a set of the allowed phase factors. Taking four allowed phase factors $+1$, -1, $+j$, $-j$ (W $= 4$) as an example, we can map each $\phi_m^{(l)}\left(k\right)$ to the allowed phase factors $\phi_m^{\prime(l)}\left(k\right)$ based on the mapping function expressed as

$$\phi_m^{\prime(l)}\left(k\right) = \begin{cases} j, & \text{if } 45^\circ \leq \phi_m^{(l)}\left(k\right) < 135^\circ \\ -1, & \text{if } 135^\circ \leq \phi_m^{(l)}\left(k\right) < 225^\circ \\ -j, & \text{if } 225^\circ \leq \phi_m^{(l)}\left(k\right) < 315^\circ \\ 1, & \text{otherwise} \end{cases}. \tag{16}$$

IV. Numerical Results

Computer simulations are discussed in this section. The simulation parameters are as follows. We consider an OFDM system with 64 subcarriers and 128 subcarriers with QPSK data symbols. In the first case, the 64 subcarriers are divided into eight subblocks consisting of eight subcarriers with four times oversampling. In the second case, the 128 sub-carries are divided into eight subblocks consisting of 16 subcarriers with four times oversampling. In order to generate the complementary cumulative distribution function (CCDF) of the PAPR, 10 000 OFDM blocks are generated randomly. Regarding the parameters used in the proposed CE algorithm, $\alpha_m^{(0)}$ is randomly chosen over $[0, 2\pi)$, $\beta_m^{(0)}$ is 10 000, the smoothing parameter γ is 0.8, $K^{\text{elite}} = 25$, $K = 200$, and the algorithm is stopped when the iteration number exceeds the predetermined value *Iter*.

For comparison, we also test 1) the simulated annealing (SA) algorithm [11] to find suitable phase coefficients for the PTS scheme; 2) the ESA [6] to search the optimal combination of phase factors. Fig. 2 shows the CCDFs for the PTS technique with the ESA, the PTS technique with the SA algorithm, the PTS technique with the proposed OCE and MCE algorithms, and the original OFDM, which is obtained directly from the output of IFFT operation for $N = 64$ subcarriers and $M = 8$ subblocks. In the ESA, four allowed phase factors $+1$, -1, $+j$, $-j$ ($W = 4$) are used, and the PAPR reduction performance is obtained by a Monte Carlo search with a full enumeration

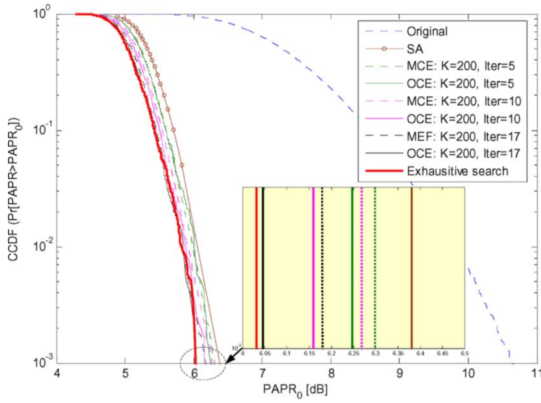

Fig. 2. CCDFs of the PAPR for various PTS techniques, with $N=64$, $M=8$, QPSK modulation, and 4 times oversampling.

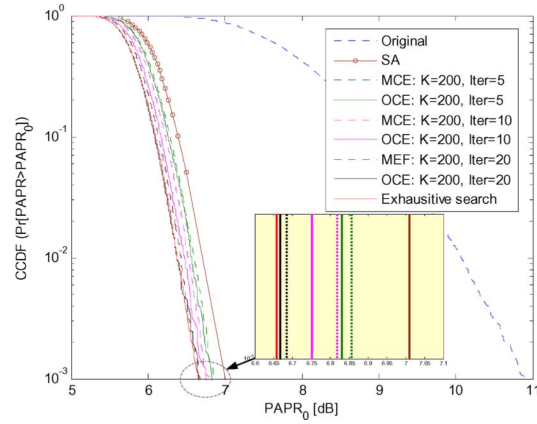

Fig. 3. CCDFs of the PAPR for various PTS techniques, with $N=128$, $M=8$, QPSK modulation, and four times oversampling.

of W^M ($4^8=65536$) phase factors. The 10^{-3} PAPR of the original OFDM signal is 10.67 dB, indicating a large PAPR. The 10^{-3} PAPR of the SA algorithm with $\text{enumeration}=1000$ is 6.38 dB. It is evident that the SA algorithm can provide the better PAPR reduction. However, when the proposed MCE method with maximum iteration value $Iter=5$, the proposed OCE method with $Iter=5$, the proposed MCE method with $Iter=10$, the proposed OCE method with $Iter=10$, the proposed MCE method with $Iter=17$, and the proposed OCE method with $Iter=17$ are employed, the 10^{-3} PAPRs reduce to 6.30 dB, 6.25 dB, 6.27 dB, 6.16 dB, 6.18 dB, and 6.04 dB, respectively. As expected, the PAPR performance improves with increasing *Iter* for both the OCE and MCE algorithms. When the maximum iteration value $Iter=17$, we can see that the proposed OCE method has approximately the same PAPR reduction performance as the ESA, whose 10^{-3} PAPR is reduced to 6.03 dB. However, the computational load of the OCE method is only 3400 $(K\times Iter=200\times 17)$, which means that the OCE method can offer good PAPR reduction while maintaining low complexity.

Fig. 3 shows the CCDFs of the PAPR of various PTS techniques for $N=128$ subcarriers and $M=8$ subblocks. It is shown in Fig. 3 that the 10^{-3} PAPRs of the original OFDM signal, the SA algorithm with $\text{enumeration}=1000$, the proposed MCE method with $Iter=5$, the proposed OCE method with $Iter=5$, the proposed MCE method with $Iter=10$, the proposed OCE method with $Iter=10$, the proposed MCE method with $Iter=20$, the proposed OCE method with $Iter=20$, and the ESA are 10.93 dB, 7.01 dB, 6.85 dB, 6.83 dB, 6.81 dB, 6.75 dB, 6.68 dB, 6.66 dB, and 6.65 dB, respectively. The results described above show that the proposed OCE method with $Iter=20$ performs almost with the same PAPR reduction as that of the ESA. However, about 16 $(65\,536/4000\approx 16)$ times less searching is required for the proposed OCE method than for the ESA. It should be noted that instead of searching the global optimal phase factor set using the OCE algorithm, the MCE algorithm aims to search the optimal combination of the allowed phase factors; therefore, the performance of the MCE algorithm is inferior to that of the OCE algorithm. However, according to Figs. 2 and 3, the performance gap between the OCE and MCE algorithms is quite small.

V. Conclusion

This paper presented a CE-based method that was used to obtain the optimal phase factor for the PTS technique to reduce the PAPR of OFDM signals. We formulated the phase factor searching of the PTS technique as a stochastic approximation problem, and we applied the CE method to search the optimal phase factor. Simulations were conducted and show that the performance of the proposed CE method provides almost the same PAPR reduction as that of the conventional ESA while maintaining low computational load.

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