Generating Capacity Reliability Evaluation Based on Monte Carlo Simulation and Cross-Entropy Methods

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***Abstract*—This paper presents a new Monte Carlo simulation (MCS) approach based on cross-entropy (CE) method to evaluate generating capacity reliability (GCR) indices. The basic idea is to use an auxiliary importance sampling density function, whose parameters are obtained from an optimization process that minimizes the computational effort of the MCS estimation approach. In order to improve the performance of the CE-based method as applied to the GCR assessment, various aspects are considered: system size, rarity of the failure event, number of different units, unit capacity sizes, and load shape. The IEEE Reliability Test System is used to test the proposed methodology, and also various modifications of this system are created to fully verify the ability of the proposed approach against both, a crude MCS and an extremely efficient analytical technique based on discrete convolution. A configuration of the Brazilian South-Southeastern generating system is also used to demonstrate the capability of the proposed CE-based MCS method in real applications.**

***Index Terms*—Cross-entropy (CE) method, generating capacity reliability (GCR), importance sampling (IS), Monte Carlo simulation (MCS), rare events, risk analysis.**

I. Introduction

In the last few decades, generating capacity reliability (GCR) indices have been very useful parameters to help planning engineers in many decisions. Various bibliographical papers have been published on this subject; see, for instance, [1]. These indices are statistical measures, which indicate the ability of the available capacity to meet the total system load. Among the analytical methods, the most popular one is known as loss of load expectation (LOLE) approach, which is a widely used technique due to its flexibility and simplicity of application [2]–[6]. This technique combines the capacity outage probability table, which models the system generation, with the system load characteristics to produce GCR indices such as loss of load probability (LOLP), LOLE, expected power not supplied (EPNS), and expected energy not supplied (EENS). The computational efficiency of these methods was considerably improved through the use of discrete convolution techniques [7], [8].

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The frequency and duration (F&D) analysis is certainly a more complete analytical approach than the LOLE, because it also provides the frequency of occurrence of an insufficient capacity condition and the duration for which it is likely to exist. These are indicated by loss of load frequency (LOLF) and loss of load duration (LOLD) indices, respectively [2]–[5], [9]. Basically, the F&D method combines, through Markov techniques, capacity and load states to obtain reserve states. Once all negative margins or failure states are identified, all aforementioned indices are evaluated. In order to make this method feasible for large real generating systems, discrete convolution techniques were extended to deal with F&D concepts [10], [11].

By avoiding combinatorial computation for practical generating systems, discrete convolution techniques have clearly increased computational efficiency of analytical methods. These methods, however, remain restricted to certain types of system conditions [12]. Conversely, Monte Carlo simulation (MCS) methods [13], [14] provide a wide-ranging reliability assessment and can be divided into nonsequential and sequential (i.e., chronological) categories. Nonsequential MCS has been an extremely useful tool in generation and/or transmission reliability evaluation of large practical systems [12], [14]–[19]. Chronological MCS methods are more powerful to evaluate complex power systems, although the computational effort is much more substantial as compared with nonsequential methods. To overcome computational burden of sequential simulations, pseudochronological MCS tools have been proposed [17]. Most recently [20], [21], population-based intelligent search techniques have also been used to solve power system reliability problems. These are interesting methods, but also have some limitations for controlling the convergence of the optimization process and dealing with large-scale systems.

Undoubtedly, MCS-based tools are extremely robust to solve power system reliability problems, particularly for large power systems. However, they may face some difficulties in dealing with rare events, e.g., to assess very small values of LOLP indices, i.e., 10^{-5}. One can argue that a very reliable generating system configuration is not a problem, and thus question the need for accurate calculation of very low levels of LOLP. However, in expansion planning studies, these values have to be accurately calculated since several reinforced configurations need to be compared. Moreover, in problems involving specific design criteria, the planner may have to measure the probability of a certain event, which may be very rare, but, *a priori*, is unknown.

In order to improve the performance of MCS tools, various variance reduction techniques are available [13]. Some of them have already been tried in power system reliability studies: see, for instance, [17] and [22]–[25]. For applications to real power systems, the efficiency of some methods can be considered relevant, others marginal. In relation to the rarity of the events, there has been little discussion in power systems reliability literature. One of the methods that showed interesting results is based on importance sampling (IS), a technique that has tremendously evolved in the last few years, with the use of optimization algorithms for selecting the parameters for the IS in an optimal way. These new algorithms are based on the concept of cross-entropy (CE) (or Kullback-Leibler) distance, which is a fundamental idea of modern information theory [26]–[28]. Some applications of this concept in power systems can already be found in [29] and [30].

This paper presents a new MCS approach based on CE method to assess GCR indices. The basic idea is to use an auxiliary IS density function, whose parameters are obtained from an optimization process. Various aspects are considered in order to improve the performance of the CE-based method, as applied to the GCR assessment: system size, rarity of the failure event, number of different units, unit capacity sizes, and load shape. The IEEE Reliability Test System (IEEE-RTS) [31] is used to test the proposed methodology, and also various modifications of this system are created to fully verify the ability of the proposed approach against both: a crude MCS and a very efficient analytical technique based on discrete convolution [10]. A configuration of the Brazilian South-Southeastern generating system is also used to demonstrate the capability of the proposed CE-based MCS method.

II. GCR Assessment

For planning studies, a power system operates successfully as long as there is sufficient power generation available to meet the load demand. Therefore, only generating units are included, and the rest of the system is assumed perfectly reliable, with unlimited capacity and without losses. In fact, losses can be approximately included.

A. Analytical Assessment

1) Generation Capacity Model: Bearing in mind the previous hypothesis, all generating units can be combined to produce an equivalent unit G, which is then combined with the total system load L. In the LOLE method, the equivalent unit G can be described by a capacity outage probability table. In the F&D method, the incremental frequency is a fundamental parameter, and it must be included in this table. Therefore, in this case, the equivalent generating unit G is described by a capacity outage probability and frequency table [10], [11].

The equivalent generating unit G can be expressed by a sum of N_G independent random variables G_k representing each generating unit. This summation can be carried out by the following recursive process, with $k = 1, \ldots, N_G - 1$:

$$G'_{k+1} = G'_k + G_{k+1} \tag{1}$$

where $G'_k = \sum G_i$ (with $i = 1, k$), and the process stops when $G'_{N_G} = G$. To illustrate the combination process, (1) will be simplified to $G = G_1 + G2$. Although the notation is tremendously simplified, the mathematical development is kept generic since G, G_1, and G_2 may represent G'_{k+1}, G'_k, and G_{k+1}, respectively.

The F&D problem can be summarized as follows: given the state capacities c, probabilities p (i.e., availabilities), and incremental frequencies q of $G_1 = \{c_1; p_1; q_1\}$ and $G_2 = \{c_2; p_2; q_2\}$, one wants to determine the same parameters for $G = \{c_G; p_G; q_G\}$. Parameters p and q are sequences of impulses associated with the sequence of state capacities c. Both sequences of impulses are equally spaced by a predefined rounding capacity increment Δ. It was fully demonstrated in [10] that parameters p and q, which characterize unit G, can be evaluated by the following convolution $(*)$ equations:

$$p = p_1 * p_2 \tag{2}$$

$$q = [p_1 * q_2] + [q_1 * p_2]. \tag{3}$$

Equations (2) and (3) are then applied to evaluate the recursive process (1), and at the end of this procedure, the parameters of the equivalent unit G are expressed by the set $G = \{c_G; p_G; q_G\}$, where, for instance, $p_G(g_i)$ is the ith term of a sequence or vector p_G, with dimension N_G, and it represents the probability associated with the generating state g_i. The discrete convolution process can be carried out by using fast Fourier transform [7] to speed up the computation.

2) Load Model: The behavior of the total system load L can be expressed by a sequence of discrete load levels defined over the period of analysis. The step of discretization can be any desirable or available time unit. Also, the load levels can be equispaced or not. From this load sequence, it is possible to build a load model described by the same parameters used in the generation model, i.e., $L = \{c_L; p_L; q_L\}$, where, for instance, $p_L(l_j)$ is a term of a vector (or sequence) p_L, with dimension N_L, and it represents the probability associated with the load state l_j. Note that, to keep the definition of incremental frequency coherent for both generation and load models, the sequence of capacities has to be ordered such that the highest capacity load state is the one associated with the minimum load level, and conversely, the smallest capacity load state is the one associated with the maximum system load.

3) Reserve Model and GCR Indices: The generating capacity model, represented by random variable G, can be combined with the load model described by random variable L to produce the power (static) reserve model R, i.e.,

$$R = G - L \tag{4}$$

with $R = \{c_R; p_R; q_R\}$, similarly to G and L.

The GCR indices, LOLP, EPNS, and LOLF, can now be evaluated from the individual reserve states of generation and load. First, (2) and (3) are used to obtain probabilities and incremental frequencies describing the states of variable R. Observe that only those parameters of state k of R, such that $c_R(r_k) = c_G(g_i) - c_L(l_j) < 0$, have to be evaluated. Considering that there are N_R of such states, then for $k = 1, N_R$

$$\text{LOLP} = \sum_k p_R(r_k) \tag{5}$$

$$\text{LOLF} = \sum_k q_R(r_k) \tag{6}$$

$$\text{EPNS} = \sum_k |c_R(r_k)| p_R(r_k). \tag{7}$$

The other three reliability indices can be computed from the ones defined earlier, i.e., $\text{LOLE} = \text{LOLP} \times T$; $\text{EENS} = \text{EPNS} \times T$; and $\text{LOLD} = \text{LOLP}/\text{LOLF}$, where T represents the period of analysis, usually 8736 or 8760 h (one year).

B. Crude MCS Model

1) State-Space Representation: The estimates of loss of load indices for generation systems are obtained based on two distinct representations: state-space and chronological modeling. State enumeration and nonsequential MCS methods are examples of state-space-based algorithms, where Markovian models are used for both equipment and load state transitions. Therefore, states are selected and evaluated without considering any chronological connection or memory. Usually, state-space-based algorithms follow three major steps: 1) select a system state (i.e., equipment availability and load level); 2) analyze the performance of the selected states (i.e., check if the total available generation is able to satisfy the associated load without violating any operating limit; if necessary, activate corrective measures such as load curtailment); 3) estimate reliability indices (i.e., LOLP, etc.); if the accuracy of the estimates are acceptable, stop; otherwise, go back to step 1).

The previous analytical model is an example of state-space representation, but all evaluations are carried through discrete convolution techniques. The following section summarizes how the nonsequential MCS, denoted by *crude*, i.e., without using IS and CE concepts, evaluates the GCR indices.

2) Estimation of GCR Indices: GCR indices can be estimated by MCS techniques as the mean over N sampled system state values $\mathbf{Y}_i$ (a vector including generating and load states) of the test function $H(\mathbf{Y}_i)$, i.e.,

$$\tilde{E}[H] = \frac{1}{N} \sum_{i=1}^{N} H(\mathbf{Y}_i). \tag{8}$$

Estimates of all the basic reliability indices can be represented by (8), depending on the definition of the test function H. The uncertainty of the estimate is given by the variance of the estimator

$$V(\tilde{E}[H]) = \frac{V(H)}{N} \tag{9}$$

where $V(H)$ is the variance of the test function. This uncertainty is usually represented as the coefficient of variation β, which is given by

$$\beta = \frac{\sqrt{V(\tilde{E}[H])}}{\tilde{E}[H]}. \tag{10}$$

Nonsequential simulation can easily provide unbiased estimates for the LOLP and EPNS indices. In this case, test functions H_{LOLP} and H_{EPNS} are given by [15]

$$H_{\text{LOLP}}(\mathbf{Y}_i) = \begin{cases} 0, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Success}} \\ 1, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Failure}} \end{cases} \tag{11}$$

$$H_{\text{EPNS}}(\mathbf{Y}_i) = \begin{cases} 0, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Success}} \\ \Delta P_i, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Failure}} \end{cases} \tag{12}$$

where $\mathbf{\Psi} = \mathbf{\Psi}_{\text{Success}} \cup \mathbf{\Psi}_{\text{Failure}}$ is the set of all possible states $\mathbf{Y}_i$ (i.e., the state space), divided into two subspaces $\mathbf{\Psi}_{\text{Success}}$ of success states and $\mathbf{\Psi}_{\text{Failure}}$ of failures states; and ΔP_i is the amount of curtailed power at the failure state $\mathbf{Y}_i$.

Nonsequential simulation can also provide unbiased estimates for the LOLF. In this case, the test function H_{LOLF} is given by [15]:

$$H_{\text{LOLF}}(\mathbf{Y}_i) = \begin{cases} 0, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Success}} \\ \Delta\lambda_i, & \text{if } \mathbf{Y}_i \in \mathbf{\Psi}_{\text{Failure}} \end{cases} \tag{13}$$

where $\Delta\lambda_i$ is the sum of the transition rates (including generating and load states) between $\mathbf{Y}_i$ and all the success states, which can be reached from $\mathbf{Y}_i$ in one transition. Obviously, for each selected state $\mathbf{Y}_i \in \mathbf{\Psi}_{\text{Failure}}$, in GCR assessment, only the units that are down have to be considered in the analysis. Another unbiased estimator for the LOLF index was proposed in [16]. Finally, the other three reliability indices (i.e., LOLE, EENS, and LOLD) can be easily evaluated.

III. GCR Assessment Via CE Method

IS is based on the idea of making the occurrence of rare events more frequent or, in other words, to speed up the simulation convergence. Technically, IS aims to select a probability density or mass function $f_{\text{opt}}(\cdot)$ different from the original, such that the sample variance is minimized. The efficiency of the IS depends on obtaining this new $f_{\text{opt}}(\cdot)$, or at least, one very close to it. The obvious problem is that the optimal change of measure or new $f_{\text{opt}}(\cdot)$ is initially unknown and generally difficult to find. The advantage of the CE method is that it provides a simple adaptive procedure for estimating the optimal, or at least close to optimal, reference parameters. This is achieved by minimizing the "distance" between the sampling $f(\cdot)$ and the optimal $f_{\text{opt}}(\cdot)$ iteratively. A particular way to measure the "distance" between two functions that has been proven useful is the so-called Kullback-Leibler divergence [26]–[28]. The idea is to adapt these concepts for solving the GCR problem. Thus, the notation used in the previous section is slightly reviewed in order to adequately formulate and solve the GCR via CE techniques.

A. Basic Concepts

Consider a system with N_C generating stations. Also, assume that the jth station (GS_j) has n_j identical and independent units, each one with a capacity C_j. The behavior of each unit in the station follows a Bernoulli distribution, with unavailability u_j. Under this assumption, each generating station follows a binomial distribution. Moreover, consider that the system load is equal to L and constant in the analysis period. The analytical problem of evaluating, for instance, the LOLP index can be described by the following equation:

$$\text{LOLP} = E_{\mathbf{u}}[H_{\text{LOLP}}(\mathbf{X})] = \sum_{\mathbf{X}_k \in \Omega} H_{\text{LOLP}}(\mathbf{X}_k) f(\mathbf{X}_k; \mathbf{n}, \mathbf{u}) \tag{14}$$

where $\mathbf{X}_k$ is a possible outcome of the state vector $\mathbf{X} = [x_1, x_2, \ldots, x_j, \ldots, x_{Nc}]$ from the space Ω, whose generic component x_j represents the number of available units at the generating station j; $\mathbf{n}$ is a parameter vector $(1 \times N_C)$, whose generic component n_j represents the total number of generating units at GS_j; $\mathbf{u}$ is also a parameter vector $(1 \times N_C)$, whose generic component u_j represents the unit unavailability; and $f(\cdot; \mathbf{n}, \mathbf{u})$ is the probability mass function associated with random vector $\mathbf{X}$.

Test function $H_{\text{LOLP}}(\mathbf{X}_k) = I_{\{S(\mathbf{X}_k)<L\}}$ is used for the LOLP index, which is equal to 1 if the event $\{S(\mathbf{X}_k) < L\}$ is true and 0, otherwise. This test function is similar to the one defined by (11), but it has been adapted to the notation used in [26] and [27]. The expression $S(\mathbf{X}_k)$ is the performance function, which is basically the summation of all available generating capacities associated with $\mathbf{X}_k$, and L represents the load level. Although the index LOLP has been used in (14), any other GCR measure could be considered based on the expressions of (12) and (13).

The probability mass function $f(\cdot; \mathbf{n}, \mathbf{u})$ represents a binomial distribution defined at $\mathbf{X}_k$ as

$$f(\mathbf{X}_k; \mathbf{n}, \mathbf{u}) = \prod_{j=1}^{N_c} \frac{n_j!}{x_j!(n_j - x_j)!}(1-u_j)^{x_j}(u_j)^{n_j - x_j}. \quad (15)$$

In order to estimate the index LOLP, nonsequential MCS can be used with (14). Considering N samples, i.e., $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N$ of generating states that follows $f(\cdot; \mathbf{n}, \mathbf{u})$ described by (15), an unbiased estimator for the LOLP index is given by

$$\widehat{\text{LOLP}} = \frac{1}{N}\sum_{i=1}^{N} I_{\{S(\mathbf{X}_i)<L\}}. \quad (16)$$

However, if the load level L and/or the unavailabilities u_j are very small, the LOLP index will be very low since the event $\{S(\mathbf{X}_i) < L\}$ will be extremely rare, and consequently, the computational effort to reach such conditions will be huge: large values of N to reach low uncertainty levels β. In order to speed up the convergence process, IS techniques must be used, and another unbiased estimator for the LOLP index is given by

$$\widehat{\text{LOLP}} = \frac{1}{N}\sum_{i=1}^{N} I_{\{S(\mathbf{X}_i)<L\}}\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \mathbf{v}) \quad (17)$$

where $\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \mathbf{v})$ is the likelihood ratio and represents the correction made in the sampling process, since the probability function $f(\cdot; \mathbf{n}, \mathbf{v})$ is being used, instead of the original $f(\cdot; \mathbf{n}, \mathbf{u})$. For this specific problem, the likelihood ratio is given by

$$\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \mathbf{v}) = \frac{f(\mathbf{X}_i; \mathbf{n}, \mathbf{u})}{f(\mathbf{X}_i; \mathbf{n}, \mathbf{v})} = \frac{\prod_{j=1}^{N_c}(1-u_j)^{x_j}(u_j)^{n_j-x_j}}{\prod_{j=1}^{N_c}(1-v_j)^{x_j}(v_j)^{n_j-x_j}}. \quad (18)$$

A distortion of the state-space probabilities is determined by the parameter vector $\mathbf{v} = [v_1, v_2, \ldots, v_j, \ldots, v_{Nc}]$, and the problem now consists of finding the best $\mathbf{v}$ that minimizes the computational effort of the MCS process.

B. CE Algorithm

Using the CE concepts described in [26] and [27], the basic idea is to find the optimal CE reference parameter vector $\mathbf{v}_{\text{opt}}$. This is achieved by the subsequent ten-step algorithm, where *step 1)* is the initialization process, *steps 2)–6)* represent the optimization CE approach, and *steps 7)–10)* are the optimized IS nonsequential MCS. Moreover, the LOLP index will be used as the illustrative test function (H and I). The other GCR indices will be discussed in Section III-C.

Step 1) Besides all values that characterized the GCR problem (i.e., generating parameters and load level L), the following additional parameters must be defined: 1) sample size N (e.g., 10 000 samples) for the optimization process [steps 2)–6)]; 2) multilevel parameter ρ (e.g., typically between 0.01 and 0.1 [26]); 3) smoothing parameter $\alpha = 1$ (only different from 1 to prevent occurrences of 0s and 1s in vector $\mathbf{v}$ [26]); and 4) maximum sample size N_{MAX} and coefficient of variation β_{LOLP} (e.g., between 1% and 5%) for *steps 7)–10)*.

Step 2) Define $\hat{\mathbf{v}}_0 = \mathbf{u}$, which means that $\hat{\mathbf{v}}_0$ is equal to the vector of the original unavailabilities of the generating system; also, set $k = 1$ (iteration counter of the CE optimization process).

Step 3) Generate a random sample $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N$ of generating states that follows $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_{k-1})$. Also, evaluate the performance function $S(\mathbf{X}_i)$ for each sampled state $\mathbf{X}_i$, and order them from the largest to the smallest, i.e., $S_{[1]} \geq S_{[2]} \geq \cdots \geq S_{[N]}$. Note that $S_{[j]}$ represents the jth-*order statistic* of the sequence $S(\mathbf{X}_1), \ldots, S(\mathbf{X}_N)$.

Step 4) Let $\hat{L}_k$ be the sample ρ-quantile of the performances: $\hat{L}_k = S_{[(1-\rho)N]}$, provided this is greater than L. Otherwise, set $\hat{L}_k = L$. Evaluate the test function for the LOLP index, i.e., $H_{\text{LOLP}}(\mathbf{X}_i) = I_{\{S(\mathbf{X}_i)<\hat{L}_k\}}$, considering all $\mathbf{X}_i$: if $S(\mathbf{X}_i) < \hat{L}_k$, then $H(\mathbf{X}_i) = 1$; otherwise, $H(\mathbf{X}_i) = 0$. Calculate vector $\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \hat{\mathbf{v}}_{k-1})$ for all $\mathbf{X}_i$ according to (18).

Step 5) Use the same sample to evaluate each element $j = 1, \ldots, N_C$ of the new reference parameter vector $\hat{\mathbf{v}}_k$ given by

$$\hat{v}_{k,j} = \frac{1}{n_j}\left[\frac{\sum_{i=1}^{N} I_{\{S(\mathbf{X}_i)<\hat{L}_k\}}\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \hat{\mathbf{v}}_{k-1})X_{ij}}{\sum_{i=1}^{N} I_{\{S(\mathbf{X}_i)<\hat{L}_k\}}\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \hat{\mathbf{v}}_{k-1})}\right]. \quad (19)$$

Equation (19) is determined by solving a stochastic optimization problem. Since the random variables involved, i.e., binomial distributions, belong to the natural exponential family, the analytical expression (19) was obtained by just adapting the concepts and proofs described in [26]–[28]. In case $\alpha \neq 1$, correct $\hat{v}_{k,j}$, as shown in [26]–[28].

Step 6) If $\hat{L}_k = L$, the CE optimization algorithm is finished at $k = K$ (final iteration), then go to *step 7)*; otherwise, increase the iteration counter as $k = k + 1$, and go back to *step 3)*.

Step 7) Now, a nonsequential MCS algorithm will be run, based on IS techniques, using the optimal vector parameter $\hat{\mathbf{v}}_K$. Therefore, set the new iteration counter $M = 0$.

Step 8) Set $M = M + 1$ and generate a sample $\mathbf{X}_M$ according to the probability mass function $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_K)$.

Step 9) Evaluate $H_{\mathrm{LOLP}}(\mathbf{X}_M) = I_{\{S(\mathbf{X}_M)<L\}}$, $\mathbf{W}(\mathbf{X}_M; \mathbf{n}, \mathbf{u}, \hat{\mathbf{v}}_k)$, and also the unbiased estimator for the LOLP index at iteration M as follows:

$$\widehat{\mathrm{LOLP}} = \frac{1}{M}\sum_{i=1}^{M} I_{\{S(\mathbf{X}_i)<L\}}\mathbf{W}(\mathbf{X}_i; \mathbf{n}, \mathbf{u}, \hat{\mathbf{v}}_K). \tag{20}$$

Step 10) Estimate the coefficient of variation $\beta_{(M)}$. If $\beta_{(M)} \leq \beta_{\mathrm{LOLP}}$ or $M \geq N_{\mathrm{MAX}}$ stop the algorithm; otherwise go back to *step 8)*. In order to make the MCS tracking process more efficient, the convergence is verified in blocks of, for instance, 1000 samples.

C. Additional Aspects

The previous GCR assessment via CE has only mentioned LOLP index. Moreover, the load was set to only one level, for instance, the peak load. In the next two sections, both aspects are duly treated.

1) Other GCR Indices: The CE-based MCS algorithm proposed in Section III-B showed the evaluation of LOLP index, and for that, the best parameter $\hat{\mathbf{v}}_K^{\mathrm{LOLP}}$ was obtained. For the other two indices, i.e., EPNS and LOLF, a similar optimization process has to be carried out to calculate the best parameter vectors, i.e., $\hat{\mathbf{v}}_K^{\mathrm{EPNS}}$ and $\hat{\mathbf{v}}_K^{\mathrm{LOLF}}$, respectively. Clearly, the test function $H_{\mathrm{LOLP}}(\mathbf{X}_i)$ and its associated output performance $I_{\{S(\mathbf{X}_i)<\hat{L}_k\}} \Rightarrow \{1 \text{ or } 0\}$ would have to be changed according to (12) and (13): for EPNS $\Rightarrow \{\Delta P_i \text{ or } 0\}$ and LOLF $\Rightarrow \{\Delta\lambda_i \text{ or } 0\}$.

The failure states that contribute to the LOLP index are the same as those that contribute to the EPNS index. While the LOLP index uses only probabilities, the EPNS captures both probabilities and the corresponding power not supplied. In relation to the LOLF index, however, it uses the states around the boundary between success and failure subspaces.

Fig. 1 illustrates the boundary line between failure and success states, the space Ω, and the regions of importance for the basic GCR indices. In case of LOLF index, any side (failure or success) could be indicated. In case of LOLP index, the magnitude of contributions decreases if samples move away from the boundary. In the case of the EPNS index, samples close to the boundary have high probabilities but, in general, small load curtailments.

The most dominant factor for all indices is indeed the probabilities, and, therefore, the failure subregion close to the boundary is the most relevant for the GCR assessment. Various tests have proven that the optimal vector parameter found for the LOLP index $\left(\hat{\mathbf{v}}_K^{\mathrm{LOLP}}\right)$ is very close to the optimal parameters obtained for EPNS $\left(\hat{\mathbf{v}}_K^{\mathrm{EPNS}}\right)$ and LOLF $\left(\hat{\mathbf{v}}_K^{\mathrm{LOLF}}\right)$ indices, i.e., $\hat{\mathbf{v}}_K^{\mathrm{LOLP}} \cong \hat{\mathbf{v}}_K^{\mathrm{EPNS}} \cong \hat{\mathbf{v}}_K^{\mathrm{LOLF}}$ [32]. In conclusion, only one optimization can be carried out [i.e., *steps 2)–6)*] based on the LOLP index and used for estimating all other GCR indices in the MCS process based on IS [i.e., *steps 7)–10)*].

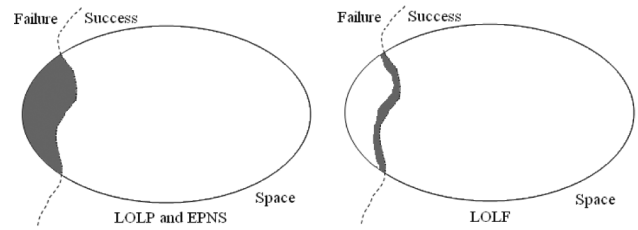

Fig. 1. Failure states and GCR indices.

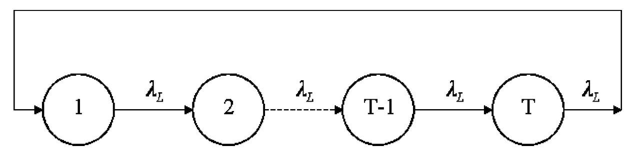

Fig. 2. Multilevel Markov load model.

2) Multilevel Load Model: Any load model is an approximation of the actual load. Its accuracy depends on the amount and quality of the available data. Markovian assumptions can always be verified not only for equipment outage times, but also for loads. If they are acceptable, the estimates based on these models will be satisfactory. The CE-based MCS algorithm proposed in Section III-B showed the evaluation of LOLP index, considering only one level L, i.e., the peak load. The proposed algorithm will be extended to deal with a multilevel load model.

Fig. 2 represents the multilevel load model to be used with the CE-based MCS approach. It describes a set of T load levels, representing, for instance, 8736 h, sequentially connected in the same chronological order as they appear in the historical sequence. The model uses a constant load transition rate $\lambda_L = 1/\Delta T$, where ΔT represents the time unit used to discretize the period T, e.g., $\lambda_L = 1$ transition per hour. Since all transition rates are the same, the load will remain, on average, ΔT hours (e.g., 1 h) at each level, and T hours (e.g., 8736 h), on average, for the total period of analysis. This concept is a particular case of the multilevel nonaggregate Markov model proposed in [17].

The problem is that the probability mass function associated with this multilevel load model does not have any specific known shape. Moreover, the optimality achieved by the CE algorithm, regarding (19), can only be ensured if the involved distributions belong to the natural exponential family. However, bearing in mind that the most relevant load state is indeed the system peak load, i.e., $L = L_{\mathrm{MAX}}$, various tests have shown that the optimal vector parameter $\hat{\mathbf{v}}_K$ obtained with the peak load [*steps 2)–6)*] can be used for estimating all other GCR indices in the MCS process based on IS [*steps 7)–10)*], considering the loads sequentially sampled from the multilevel model shown in Fig. 2, or from any other similar load model [32].

Another important problem occurs in systems with very low load factors, since most load levels or samples will be far away from the peak load, and consequently, decreasing the effectiveness of the optimal vector parameter $\hat{\mathbf{v}}_K$. In other words, failure states would still be rare, but now because of the load states, and not because of the generating parameters anymore. This problem can also be solved by using an additional parameter Φ defined as

$$\Phi = P_{\hat{\mathbf{v}}_K}\{S(\mathbf{X}_i) < \ell\}. \tag{21}$$

If one specifies the probability Φ, an estimate $\hat{\ell}$ of ℓ can be obtained by sampling N (e.g., 10 000) generating states, i.e., $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N$, according to $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_K)$, which is the distribution obtained by the CE approach [*steps 2)–6)*]. The performance function $S(\mathbf{X}_i)$ is evaluated for each state $\mathbf{X}_i$, and the results are ordered from the smallest to the largest: $S_{[1]} \leq S_{[2]} \leq \cdots \leq S_{[N]}$. The value $\hat{\ell}$ will be such that $\hat{\ell} = S_{[\Phi N]}$. Therefore, all sampled states with load levels smaller than $\hat{\ell}$ will be automatically considered as success states, and they will not be evaluated during the MCS process based on IS [i.e., *steps 7)–10)*]. This is a very simple, but very efficient procedure from the computational point of view.

The value of probability Φ depends on the rarity of the involved events, but typical values are from 0.01 to 0.04, for LOLP between 10^{-3} to 10^{-5}, and from 0.05 to 0.1, for LOLP $< 10^{-5}$. The reason why parameter Φ does work well is due to fact that a generating state will be sampled only if a load state or level greater than $\hat{\ell}$ is found, since the probability of sampling $S(\mathbf{X}_i) < \hat{\ell}$, from distribution $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_K)$, is approximately Φ. One should always keep in mind that using the Φ parameter introduces a very small inaccuracy in the estimation process, since some sampled states are promptly considered as success. This inaccuracy, however, may be insignificant if the specified value of Φ is small. As a general rule, one can use $\Phi = 0.01$, but if the load is constant, this parameter should obviously be ignored.

IV. Numerical Applications

In order to evaluate the accuracy and efficiency of the proposed CE-based MCS method, different generating systems were tested. Due to space limitation, however, only a few of them are fully reported and discussed. In all cases, the computations were performed in a MATLAB platform using an Intel Core 2 Duo 2.66 GHz processor.

A. IEEE Reliability Test System [31]

This generation system consists of 32 units with total installed capacity of 3405 MW. The load is represented by a hourly curve with 8736 chronological levels, whose peak is 2850 MW. This system is first used just to have reference values for reliability standards and CPU times. However, the IEEE RTS is not the best system to show the full capability of the proposed method as it will be discussed next.

1) IEEE RTS—Constant Load: Table I presents the results for the GCR indices considering a constant load level, i.e., 2850 MW, for the analytical, crude MCS, and CE-based MCS methods. For the analytical method, a rounding increment $\Delta = 1$ MW is used. The parameter β is specified as less than 1% to all GCR indices, and their corresponding values are shown between brackets. Usually, the EPNS and LOLF indices need more samples to converge.

All three methods for the LOLP index reach a value around 8.4×10^{-2}. The analytical, crude MCS, and CE-based MCS methods spent 0.03, 6.9, and 2.9 s, respectively. Since the involved events are not rare (i.e., 10^{-2}), there is no significant computational gain between the crude and the CE-based MCS: a speed up around 2.4.

TABLE I
IEEE RTS_1—Reliability Indices—Constant Load Model

IEEE RTS_1	LOLP	EPNS (MW)	LOLF (occ./year)	CPU Time (seconds)
Analytical	8.45778×10^{-2}	1.46936×10^{1}	1.95123×10^{1}	0.03
Crude MCS	8.42043×10^{-2} (0.73572%)	1.47239×10^{1} (0.98733%)	1.92890×10^{1} (0.92441%)	6.90
CE-MCS	8.45349×10^{-2} (0.58412%)	1.46499×10^{1} (0.41708%)	1.94612×10^{1} (0.99820%)	2.90

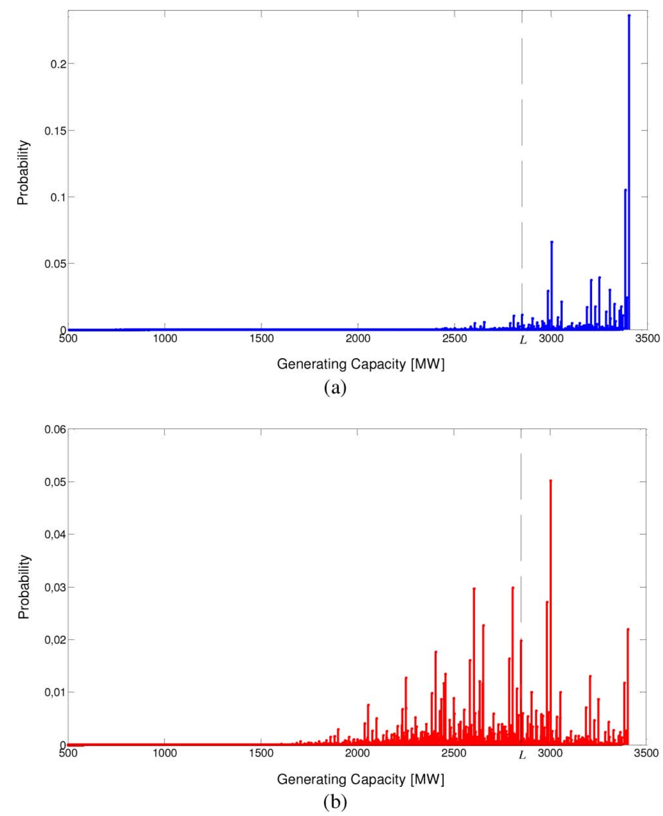

Fig. 3. Generating state probabilities.

Fig. 3(a) illustrates the generating state probabilities under the original sampling probability mass function $f(\cdot; \mathbf{n}, \mathbf{u})$, and Fig. 3(b) shows the same generating state-space, but under the optimal probability mass function $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_K)$ obtained by the optimization process [*steps 2)–6)*]. It can be concluded from Fig. 3(a) and (b) that using $f(\cdot; \mathbf{n}, \hat{\mathbf{v}}_K)$, as the sampling probability mass function, the computational performance should always be better, since the dominant failure states are being sampled more often.

An amount of CPU computational effort less than 10 s is an acceptable waiting time for any user. Conversely, waiting times of hours can really cause some impatience to users, and this is going to be illustrated next.

2) IEEE RTS—Multilevel Load: Table II presents the results for the GCR indices considering the multilevel Markovian load

TABLE II
IEEE RTS_1—RELIABILITY INDICES—MULTILEVEL LOAD MODEL

IEEE RTS_1	LOLP	EPNS (MW)	LOLF (occ./year)	CPU Time (seconds)
Analytical	1.07258×10^{-3}	1.34649×10^{-1}	2.01619	0.89
Crude MCS	1.06849×10^{-3} (0.48375%)	1.32769×10^{-1} (0.65554%)	1.99962 (0.99983%)	1440
CE-MCS (Φ=0.04)	1.07047×10^{-3} (0.41364%)	1.34107×10^{-1} (0.38734%)	2.00882 (0.99968%)	35.7

TABLE III
IEEE RTS X SCALE FACTOR—MULTILEVEL LOAD MODEL

Size	LOLP Analytical	LOLP $CE_{\beta=1}$	CPU Time			
			Ana.	$Crude_{\beta=5}$	$CE_{\beta=5}$ (Φ)	$CE_{\beta=1}$ (Φ)
1	1.07258×10^{-3}	1.06406×10^{-3}	0.89	61.0 s	2.39 (0.01)	34.6 (0.05)
5	1.24395×10^{-6}	1.21271×10^{-6}	4.70	6.75 h	4.21 (0.05)	36.6 (0.13)
10	1.06714×10^{-8}	1.06797×10^{-8}	19.4	3 mo.*	5.77 (0.05)	34.7 (0.15)
15	1.58195×10^{-10}	1.57368×10^{-10}	43.8	26 y*	5.38 (0.07)	30.6 (0.15)
20	2.69301×10^{-12}	2.69051×10^{-12}	85.0	707 y*	4.96 (0.1)	28.2 (0.2)

* These values, expressed in months (mo.) and years (y), were obtained by extrapolating the trends in the semi-log corresponding graphics.

model shown in Fig. 2, applied to the IEEE-RTS data [31]. The load factor is circa 0.6144 with a peak load of 2850 MW, and a period of 8736 h. The β parameter is also specified as 1% for all indices. In the analytical approach, $\Delta = 1$ MW is used and no truncating probabilities are specified. Note that, in this case, the proposed CE-based approach greatly outperforms the crude MCS, since the convergence was reached 40.34 times faster (speed up).

Fig. 4 illustrates the performances of both crude and CE MCS, the latter with $\Phi = 0.02$. Fig. 4(a) shows the values for the LOLP index and Fig. 4(b) shows the respective β values. The CE-based MCS approach reaches $\beta = 1\%$ in circa 8.5 s, which represent about 6.22×10^5 samples, while the conventional crude MCS reaches the same β in 332 s, using about 9.21×10^6 samples. Note that the sampling rates, i.e., samples/second, are different for both methods since $\Phi \neq 0$.

B. Modifications on the Original IEEE RTS [31]

In order to fully verify the ability of the CE-MCS against both crude MCS and analytical approach, various modifications of the original IEEE RTS systems are created. These modifications are represented by different scale factors. For instance, if a scale factor of 5 is being used (denoted as IEEE RTS_5) that means all the system main characteristics (i.e., number of units and load levels) are multiplied by this factor.

Table III presents the results for the IEEE RTS modified by 1 (i.e., the original system), 5, 10, 15, and 20 scale factors. Increasing this factor implies that the number of system states in-

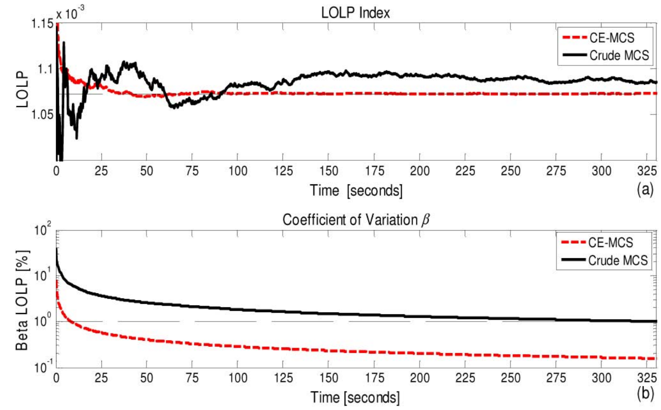

Fig. 4. Convergence process of the LOLP index.

creases, while the values of the LOLP indices decrease. This happens because the total system capacity becomes huge in comparison to a single generating unit, which means that even the failure of some of the largest units does not affect the overall reliability performance of the system significantly.

Table III shows the values for the LOLP index calculated with the analytical and CE-based MCS (with $\beta = 1\%$) methods. It also shows the CPU times obtained with: analytical (Ana.); crude MCS with $\beta = 5\%(\text{Crude}_{\beta=5})$; CE-based MCS with $\beta = 5\%$ $(\text{CE}_{\beta=5})$; and CE-based MCS with $\beta = 1\%(\text{CE}_{\beta=1})$. For the CE-based methods, different values of Φ parameter are shown between brackets.

One can observe from Table III that the results obtained with the analytical and CE-based MCS (with $\beta = 1\%$) are basically the same. Fig. 5 also illustrates these comparisons in terms of a semi-log bar diagram. Considering the increase of the scale factor from 1 to 20, the LOLP indices extend from 10^{-3} to 10^{-12}. In fact, from factor 5, the failure events in the generating system can be considered as rare (i.e., less than 10^{-5}), and the computational gains with the CE-based MCS become obvious.

Note that the performance of both, analytical and crude MCS methods, decreases when the system size increases. The analytical method is not considerably sensitive to the rarity of the involved events, but it is to the increasing number of possible states, even using an extremely efficient numerical algorithm that avoids the combinatorial curse. The opposite occurs with the crude MCS approach, since it is not sensitive to the number of system states, but it is dramatically to the rarity of the involved events.

The performance of the CE-MCS is indifferent to both problems. It does not have problems with the number of system states since it is basically an MCS, and with the rarity of the events (low probabilities), because of the optimal *distortion* applied to the unit unavailabilities. Issues concerning generating systems with low load factors can be minimized by the adequate use of Φ parameter.

Fig. 6 illustrates the performances of the methodologies when applied to the IEEE RTS with different scaling factors. It can be seen that, while the performances of both analytical and crude MCS methods get worse, the performance of the CE-MCS is practically constant, which, in this case, mostly depends on the specified values of β and Φ. It is amazing to

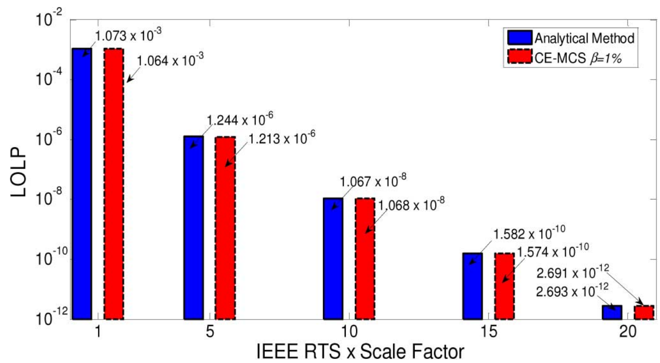

Fig. 5. LOLP values for different modifications on IEEE RTS.

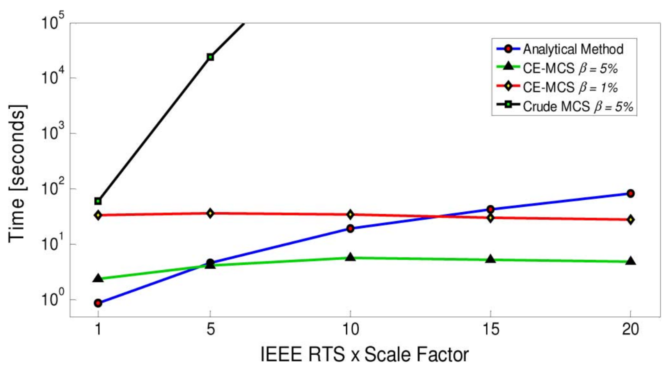

Fig. 6. Performance comparisons—IEEE RTS with different scale factors.

observe that while the crude MCS can spend hours (IEEE RTS_5) or even years (IEEE RTS_20) to conclude the reliability assessment, the CE-based MCS will spend a few seconds for any system. Moreover, the CE-based MCS becomes even faster than an extremely fast analytical method: e.g., considering $\beta = 5\%$, the breaking point is the IEEE RTS_5 (i.e., $4.70\text{s} > 4.21\text{s}$). Undoubtedly, the performance of the proposed CE-based method is outstanding.

C. Brazilian South-Southeastern System (BSS)

This study is carried out using two of the planned configurations (normal and strongly reinforced) of the BSS for the 1990s, see some details in [10]. There were 67 power generation plants: 53 hydro plants and 14 thermal plants. There were 290 units with capacities that varied from 15 up to 700 MW (ITAIPU units). The reliability of the BSS is evaluated by all three methods: the analytical, crude MCS (with $\beta = 1\%$), and the proposed CE-based MCS (with $\beta = 1\%$ and $\Phi = 0.01$ and 0.10). An hourly load model with 8736 levels is considered with a peak load of 41.2 GW. The normal configuration has an installed capacity of 42.8 GW, and the strongly reinforced one has 45.6 GW. The corresponding LOLP indices are 3.443×10^{-3} and 1.909×10^{-5}, respectively.

The CPU times for the normal configuration obtained with the analytical, crude MCS, and CE-based are, respectively, 30.2, 912.6, and 17.6 s. The CPU times for the reliable configuration obtained with the analytical, crude MCS, and CE-based are, respectively, 31.2 s, 26 h, and 31.6 s. Considering $\beta = 5\%$, the CE-based spends 3.3 s, which is much faster than the analytical approach. The huge advantage of the proposed CE-based MCS is obvious here.

V. Conclusion

MCS-based tools are extremely robust and flexible for solving power system reliability problems, particularly for large power systems. However, they may face some hurdles in dealing with rare events, e.g., to assess very small values of LOLP indices, i.e., less than 10^{-5}. Although one can say that a very reliable system configuration is not a problem, and the simulation should stop after detecting such condition, in expansion planning studies, these values have to be calculated since various reinforced configurations are being compared. In problems involving specific design criteria, the probability of certain events may have to be calculated, which may be very rare, but, *a priori*, is unknown.

This paper presented a new MCS approach based on CE method to assess GCR indices. The basic idea is to use an auxiliary IS density function, whose parameters are obtained from an optimization process. Various aspects were considered in order to improve the performance of the CE-based method, as applied to the GCR assessment.

The performance of the proposed CE-based approach was tested in several different conditions, including system sizes and rarity of failure events. The obtained results were compared with an extremely efficient analytical method based on discrete convolution technique and also with a crude MCS. The outstanding performance of the proposed CE-based method against both previous approaches was demonstrated. It is shown that the proposed approach reaches the specified convergence parameter in a few seconds, while the crude MCS may take hours, months, or even years to ensure the same accuracy. For very large generating systems, the proposed CE-based approach proves to be faster than an extremely efficient analytical method, whose the reliability assessment is restricted to applications where correlation aspects cannot be represented.

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