Cross-entropy optimisation of multiple-input multiple-output capacity by transmit antenna selection

Y. Zhang, C. Ji, W.Q. Malik, D.C. O'Brien and D.J. Edwards

Abstract: Radio channel capacity can be increased dramatically using a multiple-input multiple-output (MIMO) scheme, but at the expense of hardware complexity. An efficient approach for complexity reduction is antenna subset selection at the transmitter and/or receiver. A novel transmit antenna selection algorithm is presented using the cross-entropy optimisation method to maximise channel capacity. In contrast with the existing work, the proposed algorithm guarantees a result to within 99% of the true optimum (i.e. the maximal capacity with selected transmit antennas) with substantially low complexity. The simulation results indicate that the proposed algorithm is independent of the relationship between the selected transmit array size and receive array size. The proposed scheme has the potential to make practical MIMO systems with high performance simpler to implement.

1 Introduction

Next generation wireless communication systems are expected to provide a large channel capacity improvement over existing systems. Multiple-input multiple-output (MIMO) techniques can drastically increase the channel capacity through extra degrees-of-freedom that can be exploited by spatially or otherwise multiplexing several data streams [1]. However, the higher performance in MIMO systems comes at the expense of increased hardware and computational complexity, owing to the deployment of multiple radio-frequency (RF) chains at the transmitter and receiver end. Therefore an effective method, known as antenna selection, has been developed [2] to reduce this complexity. Using antenna selection techniques, the RF chains can optimally connect with the best subset of transmitter and/or receiver antennas. It has been demonstrated that the diversity order obtained by antenna subset selection is close to that obtained with the full set of antennas [2, 3], which strongly motivates the investigation of antenna-selection techniques.

In the earlier work, several algorithms have been developed for selecting the optimal antenna subset in MIMO wireless systems. Exhaustive search is used to minimise the average probability of error and the bit error rate with linear receivers in [4, 5], respectively. However, because the exhaustive search scheme becomes computationally prohibitive especially for MIMO systems with large arrays, some simplified selection algorithms have been presented to reduce the computational complexity. The norm-based selection (NBS) method has been suggested in [6–8] because of its very low complexity. Upper and lower bounds on the capacity with antenna selection are given in [2, 9], respectively. Finally, antenna selection approaches based on the theory of optimisation are derived in [10, 11].

In this paper, we will employ a novel antenna selection algorithm based on the cross-entropy (CE) optimisation method to investigate the transmit antenna selection problem under the following two scenarios: (1) perfect channel state information (CSI) available at the receiver only, denoted by CSIR, and (2) perfect CSI available at both the transmitter and receiver, denoted by CSIT. The difference in transmit antenna selection MIMO wireless systems under CSIR and CSIT is not only with regard to where CSI is obtained, but also where the transmit antenna selection algorithm is executed. Figs. 1 and 2 illustrate the system configuration for the proposed antenna-selection schemes.

The CE method was first presented by Rubinstein to estimate the probabilities of rare events in complex stochastic networks [12]. It was then extended to solve complicated combinatorial optimisation problems, for example the non-deterministic polynomial time hard (NP-hard) problems [13]. While most of the stochastic algorithms for combinatorial optimisation are based on local search, the CE method is a global random search procedure, with convergence proven in [14].

Thus the main contribution of this paper is to present a novel transmit antenna selection algorithm that can nearly guarantee an optimal subset of transmitter antennas without an exhaustive search. Another important contribution is that the proposed algorithm is independent of the relationship between the selected transmit and receive antenna array size because of its stochastic mechanism, which will benefit communications in some scenarios. Lastly, the simulation results validate the effective performance of the proposed transmit antenna selection algorithm.

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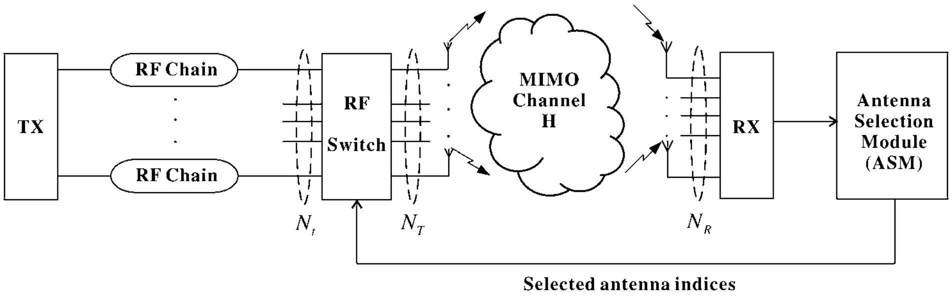

Fig. 1 *MIMO wireless system with antenna selection at the transmitter under CSIR*

The remainder of this paper is organised as follows. In Section 2, the MIMO system model is described. In Section 3, we formulate the transmit antenna selection problem as a combinatorial optimisation problem and describe the proposed antenna selection algorithm. Simulation results are provided to evaluate the performance of our algorithm in Section 4. Finally, Section 5 concludes the paper with comments on practical considerations.

2 Signal model

Consider the system block diagram of the wireless MIMO system with N_T transmit and N_R receive antennas in Fig. 1. The channel is described by an $N_\mathrm{R} \times N_\mathrm{T}$ complex matrix, denoted by $\boldsymbol{H}$. The channel is assumed to be flat Rayleigh fading and linear time invariant with additive white Gaussian noise (AWGN) at the receiver. We further assume uncorrelated fading, as the effect of correlation is beyond the scope of the current paper. Then the corresponding received signal is given by (1)

$$\boldsymbol{y} = \sqrt{\frac{E_\mathrm{s}}{N_\mathrm{T}}}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{v} \tag{1}$$

Here, $\boldsymbol{s} = [s_1, \ldots, s_{N_\mathrm{T}}]^\mathrm{T} \in \mathcal{C}^{N_\mathrm{T}\times 1}$ is the transmitted signal vector with $\mathbb{E}\{\boldsymbol{s}\boldsymbol{s}^H\} = \boldsymbol{I}_{N_\mathrm{T}}$, where $\mathbb{E}\{\cdot\}$ and $(\cdot)^H$ denote the statistical expectation and Hermitian transpose, respectively, $\boldsymbol{y} = [y_1, \ldots, y_{N_\mathrm{R}}]^\mathrm{T} \in \mathcal{C}^{N_\mathrm{R}\times 1}$ is the received signal vector, E_s is the total transmitted signal energy at one symbol time and $\boldsymbol{v} \in \mathcal{C}^{N_\mathrm{R}\times 1} \sim \mathcal{CN}(0, N_0\boldsymbol{I}_{N_\mathrm{R}})$ is an additive complex Gaussian noise vector. The channel matrix entries, h_{ij}, where $i = 1, \ldots, N_\mathrm{R}$ and $j = 1, \ldots\ N_\mathrm{T}$, represent the channel fading coefficient between the ith receive and the jth transmit antennas. For the random wireless channel, h_{ij} are independent zero-mean unit-variance complex variables.

Define the matrix

$$\boldsymbol{G} = \begin{cases} \boldsymbol{H}^H\boldsymbol{H}, & \text{if } N_\mathrm{R} \geq N_\mathrm{T} \\ \boldsymbol{H}\boldsymbol{H}^H, & \text{otherwise} \end{cases} \tag{2}$$

where the dimensions of $\boldsymbol{G}$ are $N_\mathrm{min} \times N_\mathrm{min}$ and $N_\mathrm{min} = \min\{N_\mathrm{T}, N_\mathrm{R}\}$.

The capacity of the MIMO systems under CSIR is given by [1]

$$C_\mathrm{CSIR}(\boldsymbol{H}) = \log_2 \det\left(\boldsymbol{I}_{N_\mathrm{min}} + \frac{\eta}{N_\mathrm{T}}\boldsymbol{G}\right) \tag{3}$$

where $\eta = E_\mathrm{s}/N_0$ is the average signal-to-noise ratio (SNR), $\boldsymbol{I}_{N_\mathrm{min}}$ and $\det(\cdot)$ denote the $N_\mathrm{min} \times N_\mathrm{min}$ identity matrix and the determinant operation, respectively.

Correspondingly, the capacity of the MIMO system under CSIT is given by [1]

$$C_\mathrm{CSIT}(\boldsymbol{H}) = \sum_{i=1}^{N_\mathrm{min}}\left\{\log_2\left(\frac{\eta}{N_T}\mu\lambda_i\right)\right\}^+ \tag{4}$$

where

$$\mu = \frac{N_\mathrm{T}}{N_\mathrm{min}}\left(1 + \frac{1}{\eta}\sum_{i=1}^{N_\mathrm{min}}\frac{1}{\lambda_i}\right) \tag{5}$$

and $\{\cdot\}^+$ denotes the set of positive values. The eigenvalues of $\boldsymbol{G}$ are denoted by λ_i. The difference in the antenna selection at the transmitter using (3) and (4) will be elucidated in Section 4.

3 Transmit antenna selection algorithm

In this section, we consider a MIMO system with transmit antenna selection on the following assumptions:

1. Perfect channel information at the receiver that can be transferred to the transmitter via a noise-free and instantaneous feedback channel.
2. The channel is block-fading, which means that the coherence time of the channel is long enough so that the channel statistics remain constant over the entire frame and vary in another frame independently [15].
3. There is perfect synchronisation between the different transmit and receive antennas.

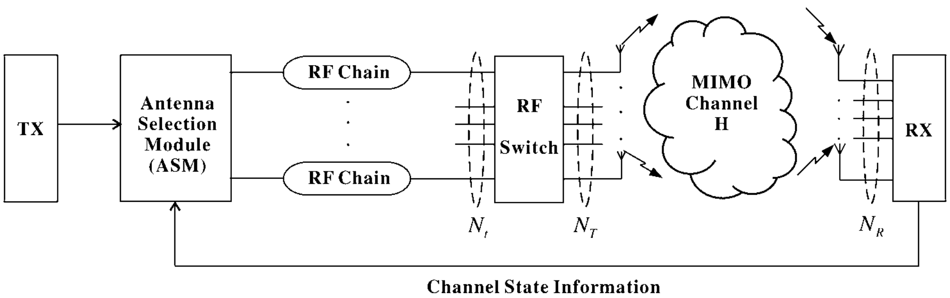

Fig. 2 *MIMO wireless system with antenna selection at the transmitter under CSIT*

3.1 Problem statement

We denote the number of the available transmit antennas and selected transmit antennas by N_T and N_t, respectively, where $N_t \leq N_T$. Moreover, we denote the set of all $Q = \begin{pmatrix} N_T \\ N_t \end{pmatrix}$ antenna subset as $\Omega = \{\omega_1, \ldots, \omega_Q\}$, where $\begin{pmatrix} x \\ y \end{pmatrix}$ denotes the binomial coefficient, $x!/(y!(x-y)!)$. Moreover, the indicators of the selected subset of transmit antennas can be denoted by

$$\omega_q = \{I_{\alpha_i}\}_{\alpha_i=1}^{N_T}, \quad I_{\alpha_i} \in \{0, 1\}, \quad q = 1, 2, \ldots, Q \tag{6}$$

where α_i is the index of the column of $\boldsymbol{H}$, and the indicator function I_{α_i} indicates whether the α_i^{th} column of $\boldsymbol{H}$ is selected or alternatively the α_i^{th} transmit antenna is selected. For example, if the first, fourth, fifth and eighth transmit antenna are selected out of eight transmit antennas, then ω_q will be equal to $\{1, 0, 0, 1, 1, 0, 0, 1\}$. The maximal CSIR capacity associated with the antenna selection using (3) is described as

$$C_{\text{CSIR}}(\boldsymbol{H}^{(\omega_q)}) = \max_{\omega_q \in \Omega} \log_2 \det\left(\boldsymbol{I}_{N_{\min}}^{(\omega_q)} + \frac{\eta}{N_t}\boldsymbol{G}^{(\omega_q)}\right) \tag{7}$$

where $\boldsymbol{I}_{N_{\min}}^{(\omega_q)}$ is the $N_{\min}^{(\omega_q)} \times N_{\min}^{(\omega_q)}$ identity matrix

$$\boldsymbol{G}^{(\omega_q)} = \begin{cases} (\boldsymbol{H}^{(\omega_q)})^H \boldsymbol{H}^{(\omega_q)}, & \text{if } N_R \geq N_t \\ \boldsymbol{H}^{(\omega_q)}(\boldsymbol{H}^{(\omega_q)})^H, & \text{otherwise} \end{cases} \tag{8}$$

and $\boldsymbol{H}^{(\omega_q)}$ is composed of columns selected from $\boldsymbol{H}$ indexed by ω_q, and $N_{\min}^{(\omega_q)}$ denotes $\min\{N_R, N_t\}$.

The maximal CSIT capacity associated with the antenna selection using (4) is given by

$$C_{\text{CSIT}}(\boldsymbol{H}^{(\omega_q)}) = \max_{\omega_q \in \Omega} \sum_{i=1}^{N_{\min}^{(\omega_q)}} \left\{\log_2\left(\frac{\eta}{N_t}\mu^{(\omega_q)}\lambda_i^{(\omega_q)}\right)\right\}^+ \tag{9}$$

where

$$\mu^{(\omega_q)} = \frac{N_t}{N_{\min}^{(\omega_q)}}\left(1 + \frac{1}{\eta}\sum_{i=1}^{N_{\min}^{(\omega_q)}} \frac{1}{\lambda_i^{(\omega_q)}}\right) \tag{10}$$

It is generally intractable to obtain an exact closed-form solution for (7) and (9) [10–11]. Therefore we model (7) and (9) as a combinatorial optimisation problem

$$\omega_q^* = \arg\max_{\omega_q \in \Omega} C_{\text{CSIR/CSIT}}(\boldsymbol{H}^{(\omega_q)}) \tag{11}$$

where ω_q^* denotes the optimum of the objective function, $C(\boldsymbol{H}^{(\omega_q)})$. After transforming (7) and (9) into an optimisation problem (11), complex closed-form analysis is not necessary. Moreover, because the CE method is a stochastic search technique in nature, the only parameter needed for the proposed transmit antenna selection algorithm is the vector $\boldsymbol{p} = [p_1, \ldots, p_{N_T}]$ whose entries, p_i indicate the probability of ith transmit antenna to be chosen. This can be adaptively adjusted to make the algorithm move towards a better solution with higher capacity performance, and after a number of iterations, the algorithm approaches the optimum point, ω_q^*. More details about parameter vector $\boldsymbol{p}$ will be given in the following section. Owing to its stochastic feature, the antenna selection algorithm with CE method is independent of the mutual relationship between the number of selected transmit and receive antennas, which will benefit communications in some scenarios. Hence, the CE method is adapted to solve (11), and is termed as cross-entropy antenna selection (CEAS) in the rest of this paper.

3.2 Proposed selection algorithm (CEAS)

For the sake of completeness, in this section we briefly review some of the key features of the CE optimisation that are relevant to our transmit antenna selection algorithm. For details, refer to [16].

The idea of the CE method is to associate a stochastic estimation problem to the optimisation problem (11). Let us define a collection of indicator functions $\{I_{\{C(\omega_q)\geq r\}}\}$ in the solution space Ω for various thresholds (or levels) $r \in \{C(\omega_q)\colon \omega_q \in \Omega\}$ and a family of Bernoulli probability density functions

$$f(\omega_q, \boldsymbol{p}) = \prod_{i=1}^{N_T} p_i^{I_{\{\alpha_i\}}(\omega_q)}(1-p_i)^{1-I_{\{\alpha_i\}}(\omega_q)} \tag{12}$$

which is parameterised by the vector $\boldsymbol{p}$. For a given probability $\boldsymbol{v}$, we associate (11) with the following stochastic estimation

$$\ell(r) = \mathbb{P}_{\boldsymbol{v}}(C(\omega_q) \geq r) = \sum_{\omega_q \in \Omega} I_{\{C(\omega_q)\geq r\}} f(\omega_q, \boldsymbol{v})$$

$$= \mathbb{E}_{\boldsymbol{v}}[I_{\{C(\omega_q)\geq r\}}] \tag{13}$$

where l is the probability for the case that $C(\omega_q) \geq r$ and $I_{\{C(\omega_q)\geq r\}}$ is defined by

$$I_{\{C(\omega_q)\geq r\}} = \begin{cases} 1, & \text{if } C(\omega_q) \geq r \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

The CE method uses an adaptive importance sampling algorithm to update the parameter $\boldsymbol{p}$ of the importance distribution $f(\cdot, \boldsymbol{p})$ such that the Kullback–Leibler divergence (so-called cross entropy) between the optimal importance function $g^*(\omega_q) = (I_{\{C(\omega_q\geq r)\}f(\omega_q,\boldsymbol{v})}/\ell)$ and f is minimal. To minimise this Kullback–Leibler divergence is equivalent to solving the maximisation problem [12]

$$\max_{\boldsymbol{p}} \int_{\Omega} g^*(\omega_q) \ln f(\omega_q, \boldsymbol{p})\, d\omega_q \tag{15}$$

which is equivalent to the following stochastic program [13,14]

$$\max_{\boldsymbol{p}} \hat{D}(\boldsymbol{p}) = \max_{\boldsymbol{p}} \frac{1}{N}\sum_{n=1}^{N} I_{\{C(\omega_q^{(n)})\geq r\}} \ln f(\omega_q^{(n)}, \boldsymbol{p}) \tag{16}$$

$$= \max_{\boldsymbol{p}} \frac{1}{N}\sum_{n=1}^{N} I_{\{C(\omega_q^{(n)})\geq r\}} \times \ln\left(\prod_{i=1}^{N_T} p_i^{I_{\alpha_i}(\omega_q^{(n)})}(1-p_i)^{1-I_{\alpha_i}(\omega_q^{(n)})}\right)$$

where $\omega_q^{(n)}$ are samples drawn from $f(\omega, \boldsymbol{v})$; $\hat{D}$ is the unbiased estimator of D, which is the Kullback–Leibler divergence defined by [17] and N is the total number of samples used to obtain the estimation of D. It is straightforward to show that the $(\partial^2\hat{D}(\boldsymbol{p}))/\partial p_i^2 < 0$ (for $i = 1, 2, \ldots, N_T$). So we can find the maximum of $\hat{D}(\boldsymbol{p})$ by setting $(\partial\hat{D}(\boldsymbol{p}))/\partial p_i = 0$, and consequently obtain the update rule as follows

$$p_i = \frac{\sum_{n=1}^{N} I_{\{C(\omega_q^{(n)})\geq r\}} I_{\{\alpha_i\}}(\omega_q^{(n)})}{\sum_{n=1}^{N} I_{\{C(\omega_q^{(n)})\geq r\}}} \quad \text{for } i = 1, 2, \ldots, N_T \tag{17}$$

Equation (17) is iteratively used with the aim of generating a sequence of increasing threshold value $r^{(0)}$, $r^{(1)}$, until convergence to the global optimum $C_{\mathrm{CSIR/CSIT}}(\boldsymbol{H}(\omega_q^*))$ (or to a value close to it) is achieved. At each iteration t, a new vector $\boldsymbol{p}^{(t)}$ is used to draw a set of new samples, which provide a better value of $r^{(t)}$. The vector $\boldsymbol{p}^{(t)}$ is then updated by these samples. This process stops when either the global optimum ω_q^* is reached or the vector $\boldsymbol{p}$ converges to a certain value within the defined bounds.

3.3 Transmit antenna selection algorithm

Choosing N_t out of N_T transmit antennas leads to a total of $N_T!/(N_t!(N_T - N_t)!)$ possible combinations for selection at the transmitter. The most direct approach is to obtain the optimal antenna subset by exhaustive search. However, this method will become computationally expensive for MIMO wireless systems with a large array size. In order to mitigate the computational burden, a new antenna-selection scheme called CEAS was presented. For easier understanding, the flow of the CEAS algorithm is shown in Fig. 3.

Note that using the approach adopted in [13], a smoothing factor, λ, was introduced to prevent the occurrence of 0s and 1s in the parameter matrix $\boldsymbol{p}$. Then the updating procedure is as follows

$$\boldsymbol{p}^{(t)} = \lambda^* \boldsymbol{p}^{(t)} + (1 - \lambda)^* \boldsymbol{p}^{(t-1)} \tag{18}$$

When $\lambda = 1$, we have the original updating formulation.

Table 1: 10% Outage capacity of transmit antenna selection with three iterations against various selection schemes with $N_R = 3$ $N_t = 4$ and $N_T = 8$ at $\eta = 10$ dB under CSIR and CSIT

Antenna selection scheme	Capacity, bps/Hz under CSIR	Capacity, bps/Hz under CSIT
CEAS selection algorithm	9.3	10.4
optimal selection	9.4	10.5
random selection	7.2	8.4
worst selection	5.7	6.9

4 Simulation results

All simulation results are obtained over 10 000 independent MIMO channel realisations. Moreover, in order to validate the efficiency of our algorithm, we employ an exhaustive search over all possible $\binom{N_T}{N_t}$ antenna subsets to obtain an optimal solution, which will be used to compare with the result of our CEAS algorithm. The following simulations are based on the two different scenarios, namely CSIR and CSIT.

Initially, 10 000 channel matrices $\boldsymbol{H}$ are generated randomly based on the assumption in Section 2, which will not change in the whole simulation for fair comparisons. A 10% outage capacity against various selection schemes with $\eta = 10$ dB under CSIR and CSIT is listed in Table 1. The capacity with the optimal selection, CEAS algorithm, random selection and worst selection are compared. From the simulation results, we find that the results with the CEAS algorithm are nearly the same as the optimal ones using the exhaustive search method. For the same problem, there are $N \times t = 12 \times 3 = 36$ computation loops for our algorithm to obtain the optimal result, whereas more than 700 computation loops are required for

The CEAS Algorithm

Transmit Antenna Selection based on the Cross Entropy Method

Step 1: Start with an initial probability vector $\mathbf{p}^{(0)} = \{p_i^{(0)}\}_{i=1}^{N_T}, p_i^{(0)} = \frac{1}{2}$. Set the iteration counter $t := 1$;

Step 2: Generate samples $\{\omega_q^{(n)}\}_{n=1}^{N}$ from the density function $f(;, \mathbf{p}^{(t-1)})$ where ω_i are samples defined by (6) and N is the number of samples used to evaluate the objective functions (7) and (9).

Step 3: Calculate the performance functions $\{C(\omega_q^{(n,t)})\}_{n=1}^{N}$ and order them from largest to smallest, $C^{(1)} \geq \cdots \geq C^{(N)}$. Let $r^{(t)}$ be $(1-\rho)$ sample quantile of the performances: $r^{(t)} = C^{(\lceil(1-\rho)N\rceil)}$.

Step 4: Update the parameter $\mathbf{p}^{(t)}$ via

$$p_i^{(t)} = \frac{\sum_{n=1}^{N} I_{\{C(\omega_q^{(n,t)})\geq r^{(t)}\}} I_{\{\alpha_i\}}(\omega_q^{(n,t)})}{\sum_{n=1}^{N} I_{\{C(\omega_q^{(n,t)})\geq r^{(t)}\}}}$$

where the probability entries, $p_i[i = 1, \cdots, N_T]$, represent the probability of the i^{th} transmit antenna to be chosen.

Step 5: If stopping criterion is satisfied, then stop; otherwise set $t := t + 1$ and go back to step 2. The stopping criterion we used here is either the predefined number of iteration is required or the probability of the transmit antenna to be chosen is equal to 1.

Fig. 3 *CEAS algorithm*

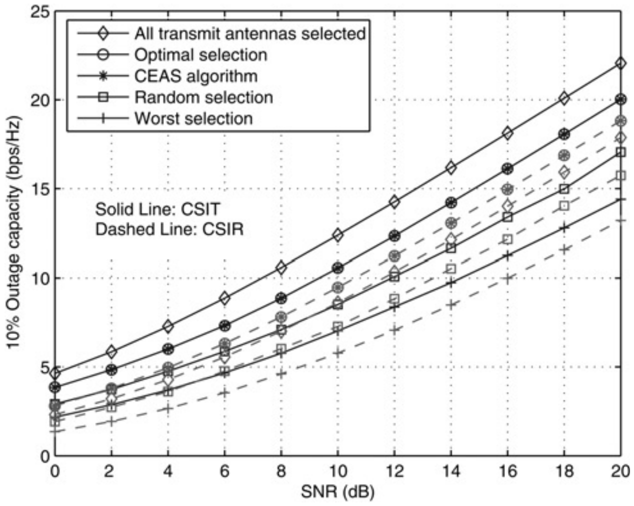

Fig. 4 *10% outage capacity against SNR with $N_R = 3$, $N_t = 4$ and $N_T = 8$*

[11]. For the antenna configuration $N_{\mathrm{R}} = 3$, $N_{\mathrm{t}} = 4$ and $N_{\mathrm{T}} = 8$, the parameters of the CEAS algorithm are set as follows: the number of samples $N = 3$, the number of iterations $t = 3$, sample quantile coefficient $\rho = 0.9$ and smoothing factor $\lambda = 0.95$. Simulations have shown that the proposed transmit antenna selection algorithm with these parameters can guarantee a result to within 99% of the optimum obtained by exhaustive search. Note that the number of samples (N) changes with various antenna configurations. Generally, the higher the cardinality of $Q = \binom{N_{\mathrm{T}}}{N_{\mathrm{t}}}$, the more the number of samples (N), given a fixed number of iterations.

Fig. 4 shows the 10% outage capacity against SNR. It can be seen that the outage capacity achieved by the CEAS algorithm is nearly the same as that using the optimal selection method for a wide range of SNR values. The CEAS algorithm is therefore superior to previously proposed algorithms such as NBS that fails at high SNR [16]. The superior feature would enable the CEAS algorithm to perform well irrespective of SNR, which may vary to a great extent in certain propagation scenarios. Moreover, our simulations confirm that the antenna selection capacity under CSIT is higher than the capacity under only CSIR. The capacity

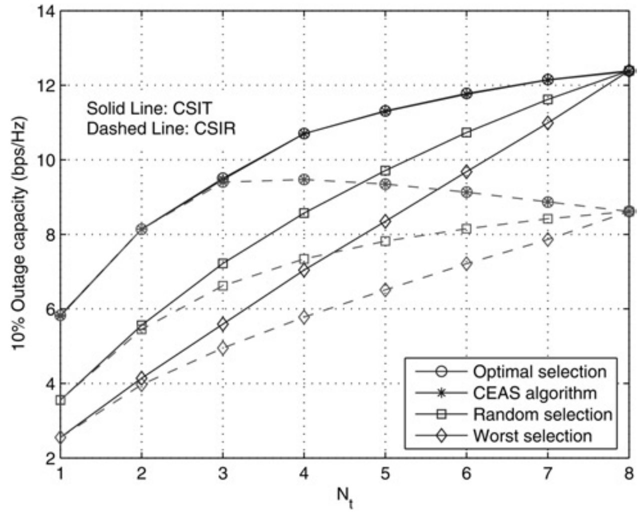

Fig. 6 *10% outage capacity against N_t with $N_R = 3$, and $N_T = 8$ at $\eta = 10$ dB*

improvement, however, comes at the cost of increased system complexity arising from the requirement for a feedback link and optimal power allocation unit at the transmitting antenna array.

Fig. 5 depicts the cumulative distribution function (CDF) against capacity at $\eta = 10$ dB. We can observe that CEAS algorithm performance coincides with that of optimal selection and the performance improves greatly compared with random selection.

Fig. 6 presents an insight into the relationship between the 10% outage capacity and the number of selected transmit antennas, N_{t}. It can be seen from the figure that when using the CEAS algorithm under both CSIR and CSIT the outage capacity increases with N_{t} until $N_{\mathrm{t}} = N_{\mathrm{R}}$. After that, there is a gap between the outage capacity of CSIR and that of CSIT; the gap widens with the increase of N_{t}. Moreover, when using the CEAS algorithm under of CSIR the outage capacity decreases at a very slow rate after $N_{\mathrm{t}} > N_{\mathrm{R}}$. When $N_{\mathrm{t}} > N_{\mathrm{R}}$, the additional transmit antennas may be destructive in some situations as also observed in [18]. Moreover, from the results we can validate that the performance of the antenna selection algorithm with CE method is independent of the relationship between the selected transmit and the receive antenna array size.

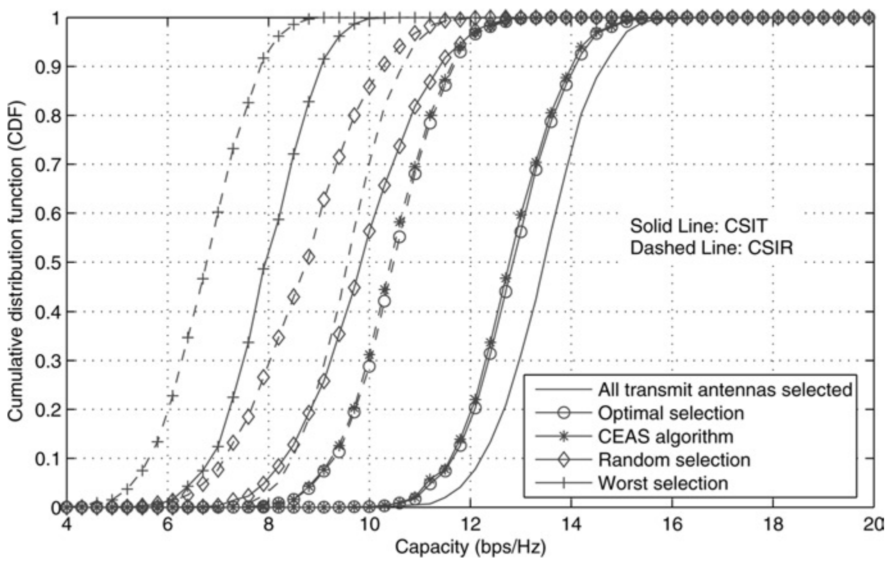

Fig. 5 *Cumulative distribution function (CDF) of capacity with $N_R = 3$, $N_t = 4$ and $N_T = 8$ at $\eta = 10$ dB*

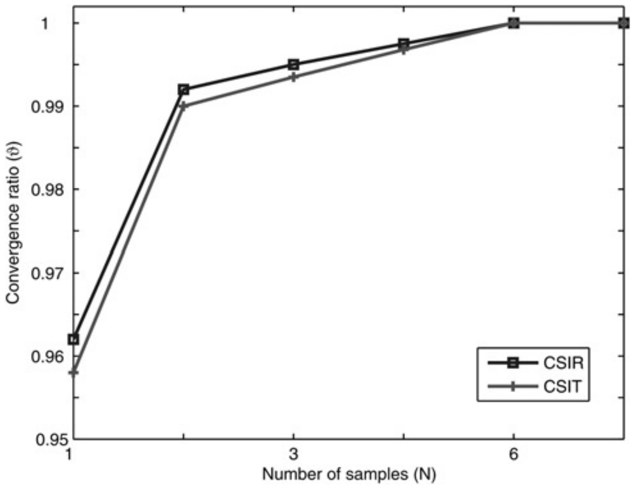

Fig. 7 *Convergence ratio (ϑ) against number of samples (N) with three iterations at $N_R = 3$, $N_t = 4$, $N_T = 8$ and $\eta = 10$ dB*

Table 2: Complexity of antenna selection algorithms where L is the number of selected transmit antennas N_t, M is the number of total antennas either N_T or N_R, N and t are the number of samples and iterations used by CEAS algorithm

Selection scheme	Complexity order O	$L = 4$ $M = 8$
exhaustive search	$O((M!/L!(M! - L!))\, ML^2)$	8960
sub-optimal selection algorithm [19]	$O(M^4)$	4096
convex optimization method [10]	$O(M^{3.5})$	1448
fast antenna selection algorithm [16]	$O(LM^2)$	256
geometrical approach [20]	$O(LM^3)$	2048
CEAS algorithm	$O(NtML^2)$	1152

In order to validate the convergence of the CEAS algorithm by simulation, the iteration order of the CEAS has been predefined as three. In this case, the relationship of the convergence ratio (ϑ) and the number of samples can been seen in Fig. 7. From this figure it can be seen that the convergence ratio increases with the number of samples when the number of iterations is fixed. Moreover we find that the CEAS algorithm under CSIR and CSIT converges to optimum point when $N = 12$. In fact, from the simulation results we find that given a sufficient number of samples or iterations, the CEAS can obtain the optimum. The theoretical convergence proof of the CE method can be found in detail in [14].

A detailed complexity comparison among the CEAS algorithm and other algorithms is shown in Table 2. From this table it can be seen that the complexity of the CEAS algorithm mainly relates to the number of samples (N) and the number of iterations (t). Compared with other algorithms, the CEAS algorithm generally exhibits much lower complexity. As compared to [16], CEAS has a higher complexity; however, a detailed comparison reveals that CEAS approaches the optimum point with comparable complexity.

5 Conclusion

In this paper, we have presented a novel transmit antenna selection algorithm to maximise the channel capacity based on the CE method. Simulations demonstrate that the CEAS algorithm can guarantee a result to within 99% of the true optimum with very fast convergence, and is independent of the relationship between the selected transmit and the receive antenna array size, which will benefit communication in some scenarios. Moreover, the simulations also validate that the CEAS algorithm performs reliably under a variety of SNR conditions. The CEAS algorithm can facilitate the practical implementation of reduced-complexity transmit antenna selection MIMO wireless systems.

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